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MAUSUMI DAS
Email: mausumi@econdse.org
Delhi School of Economics
University of Delhi

SUBRATA GUHA
Email: subrataguha@hotmail.com
Centre for Economic Studies & Planning,
Jawaharlal Nehru University
New Delhi 110067

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Department of Economics, Delhi School of Economics

What Do Teachers Do? Teacher Quality vis–a-vis Teacher Quantity in a Model of Public Education and Growth

Mausumi Das*
Delhi School of Economics

Subrata Guha†
Jawaharlal Nehru University

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Abstract

This paper analyses the contribution of teachers in a public education system and its implication for growth. We focus exclusively on two teacher-specific inputs (teacher quality and teacher quantity), and two student-specific inputs (ability and effort). We argue that all these factors enter separately in the education technology and therefore have differential impact of the process of human capital formation. In a public education system where teachers' remunerations are paid by the goverment and financed by taxation, for any given amount of government revenue, there exists a trade-off between teacher quality and teacher quantity. At the same time, the imposed tax rate has an impact on the effort choice of an agent. Thus human capital formation and growth in the model depends on a complex interaction between teacher quality, teacher quantity, student ability and student effort. In this context we discuss the optimal education policy as well the optimal taxation policy of the government.

Keywords: Public Education, Quality of Education, Growth
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*Department of Economics, Delhi School of Economics, University of Delhi, Delhi 110007, India. Email: mausumi@econdse.org

†Centre for Economic Studies & Planning, Jawarlal Nehru University, New Delhi 110067, India. Email:subrataguha@hotmail.com

1 Introduction

This paper examines the contribution of teachers in the process of human capital formation and growth. Modern growth theory emphasizes the role of human capital in economic development and growth. Accordingly education and schooling constitute important components of development strategies in almost all countries. But schooling per se does not necessarily lead to higher growth (Benhabib and Spiegel, 1994; Pritchett, 2001; Easterly, 2001). Obviously the quality of schooling matters. There exist significant differences in the quality of schooling in the developed vis-a-vis the developing world, which evidently have impacted upon their respective growth trajectories. As Hanushek and Woessmann (2007) observe: "Most people would, in casual conversation, acknowledge that a year of schooling in a school in a Brazilian Amazon village was not the same as a year of schooling in a school in Belgium. ... The data suggest that the casual conversation may actually tend to understate the magnitude of differences. ..(I)gnoring quality differences significantly distorts the picture about the relationship between education and economic outcome." And yet there are very few works in the growth theory literature that take the quality factor explicitly into account. This paper is an attempt towards this direction.

We focus explicitly on quality of schooling in a public education system and analyse its impact on human capital formation and growth. Quality of schooling has many different dimensions - some of which are teacher-specific, some of which are related to the school environment. Each of these plays a distinct role in the learning process of a student. Here we concentrate on two teacher-specific inputs: the teacher-student ratio and the quality of teaching. The teacher-student ratio (which we call 'teacher quantity') signifies how much personal attention the teacher can give to a student. The quality of teaching on the other hand refers to proficiency of the teacher in imparting knowledge through class room instructions and/or the ability to communicate with the large body of students in a class room. While this second aspect of the formal schooling process is not directly measurable, we infer that a better qualified (or better trained) teacher would in general be able to convey the teaching material to the student more lucidly. Thus this particular aspect of schooling quality is captured in our model by the average level human capital of teachers. We provide a theoretical framework relating these two aspects of quality of schooling to growth in a public education regime.

We develop an endogenous growth model where quality of schooling affects the skill level acquired by the future generation. Quality of schooling improves if the number of teachers (per student) goes up and if better quality teachers are employed. But in order to attract better quality teachers from their alternative professions, the government has to pay higher salaries. Thus for any given amount of total government expenditure on schooling, there is a trade-off: the government has to decide whether to go for higher teacher-quality or teacher-quantity. In this context we derive the optimal education policy of the government.

This paper also addresses a growing concern among the policymakers about the declining trend in the avearge aptitute of teachers. The popular belief seems to be that lucrative employment opportunities elsewhere have attracted higher ability agents away from teaching, thereby reducing the quality of schooling. The falling aptitute of teachers over the years is well-documented in the emprical literature (see, for example, Bollou and Podgursky (1995); Corcoran et al (2004a, 2004b); Hoxby and Leigh (2004)). However in view of the above mentioned trade-off.between teacher-quality and teacher-quantity, we show that under certain conditions, it may not actually be optimal for the governement to employ the best ability agents to the teaching profession, even from the perspective of long run growth.

Apart from the efficiency aspect of a given budgetary allocation on schooling (and the assocaited teacher quality-teacher quantity trade off), the total public expenditure on education itself is endognous and depends on the choice of tax rate. Thus the tax rate is an additional instrument that the government can use to influence schooling outcome and the consequent growth process. But this entails a different kind of trade off: an increase in the tax rate eases the revenue constraint of the government and thus improves the overall quality of schooling; at the same time it lowers the incentives of the students to exert effort (since a part of the product of this effort would be taxed away). Therefore there exists an optimal level of taxation that would maximise growth. In a heterogenous household framework, this optimal tax rate may or may not conincide with the most prefered tax rate of an agent. Thus we derive the growth-maximising tax rate for the economy and compare it with the tax rate which is decided by majority voting.

There exists a large body theoretical work pertaining to public education and growth (e.g., Glomm and Ravikumar (1992, 2001); Eckstein and Zilcha (1994); Benabou (1996); Zhang (1996); Blankenau and Simpson (2004); Boldrin (2005); Viaene, J.M and Zilcha, I. (2009)). However, most of these models focus on overall public expenditure on education and do not differentiate between quality and quantity. In contrast, we usively on the quality of schooling - in particular and the teacher-student ratio and teacher quality and analyse how these factors together impact on the learning process and the consequent human capital formation. In this sense our model complements this existing body of theoretical work.

The quality-quantity consideration has been explicitly taken in account in Tamura (2001) and our framework is closely related to that of Tamura. In fact we extend Tamura's work to an economy with hetergenous agents and in this context discuss the optimal policy choice of recruiting teachers in the public education institutions. In particular, we analyse the skill profile of the teachers, i.e., whether they chosen from the top end of the skill distribution, or the bottom end, or somewhere from the middle.

The paper that comes closest to our work is that of Gilpin and Kaganovich (2012) who analyse a similar quantity-quality trade off in a two-tiered education system, comprising of basic education and college education. However there are two crucial differences between our work and that of Gilpin-Kaganovich. In Gilpin and Kaganovich, there exists a fixed pre-requisite human capital threshold for entering college education which generates increasing returns to skill formation for higher ability people. This implies that only the relatively high ability people will go for skill formation. This feature along with the assumption that only a college-educated can enter the teaching profession automatically ensures a certain minimum quality of teachers (in terms of ability), which, to our mind, undermines the implied quality-quantity trade-off. In other words, it rules out the possibility that under certain scenario the government might find it optimal to employ teachers from the botton rung of the ability distribution. In contrast, in our model there is no such invisibility in skill formation, which allows us to get starker results in terms of teacher-quality and teacher-quantity choices of the government. There is a second important distinction between the two papers, which arises due to different specifications of the human capital formation function. In Gilpin and Kaganovich, the only student-specific input that enters into human capital formation is ability. We, on the other hand, allow for two student-specific inputs: ability and effort. This allows us to capture an interesting interplay bewteen ability and effort choices which may generate a non-monotonic relatioship between ability and human capital formation. For example, under certain incentive structure, a low ability agent may exert more effort and acquire same or higher level of human capital than a high ability agent, which in turn will have implications for the ability and/or skill profile of the selected teachers.

At a broader level our work contributes the literature on public spending and growth (e.g., Barro (1990); Alesina and Rodrik (1994)). Our model can be interpreted as an extension of this framework by introducing a quality-quantity choice in the provision of the public input.

The structure of the paper is as follows. The next section describes the general framework of the model. Section 3 elaborates upon the education policy. Section 4 describes the human capital dynamics and growth. Section 5 decribes the optimal taxation policy of the government.It also compares various optimal tax rates when objective functions differ. Section 6 concludes the paper.

2 The Model

2.1 General Framework

Time is discrete, represented by $t = 0, 1, 2, ...$ At any point of time the economy is populated by two successive overlapping generations of dynasties. Each generation consists of a continuum of population of measure one.

Each agent is born with some innate ability[1] which varies across agents within a cohort. An agent knows his own ability but it is not observable to others - although the distribution of ability is known to all. We assume that innate ability within a cohort is uniformly distributed over the unit interval $[0, 1]$. Thus agents belonging to the same generation can be indexed by their inherent ability factor, x , such that $x \in [0, 1]$. Also, innate ability is i.i.d. accross generations which implies that parental ability does not directly impact on childrens' ability.

The life cycle of a representative agent of any cohort is as follows. The agent lives exactly for two periods - defined for convenience as childhood and adulthood, and has exactly one offspring born to her during adulthood. During her childhood the agent consumes nothing and only exerts effort in acquiring education/skill. Upon adulthood, she works and earns a certain wage income (depending on the skill level acquired by her during childhood) - of which a part is contributed towards the education (publicly provided) of the next generation and the rest is consumed. The agent dies at the end of this period.

2.2 Preferences

Agents within a generation and across generations have identical preferences. An agent derives positive utility from own adulthood consumption, denoted by c, and from the contribution made towards next generation's education, denoted by b. The latter can be thought of as a proxy for educational bequest - although the entire amount may not be spent in educating her own child.[2] The agent also derives negative utility from efforts exerted during

[1] The term 'ability' does not necessarily mean intelligence or merit. It could represent factors like patience, tenacity, motivation, ambition or any other individual-specific factor which determines educational achievement - over and above effort.

[2] This is equivalent to the 'warm glow' bequest assumption in Galor-Zeira (1993) and Banerjee-Newman (1993). Under a private education regime the agent herself would have optimally chosen the exact amount to be received by her progeny as educational bequest. However with public provision of education, a constant

childhood in acquiring education. Consider a young agent born at time t who exerts an effort level e_t during childhood in acquiring education, enjoys an adulthood consumption of c_{t+1}, and contributes b_{t+1} towards the public education system. The lifetime utility of this agent is represented by the following quasi-linear utility function:

$$U \equiv (c_{t+1})^{\epsilon} (b_{t+1})^{1-\epsilon} - e_t; \quad 0 < \epsilon < 1. \tag{1}$$

2.3 Production

A single final commodity using human capital/skill (H). Technology for final good production is standard AK type:

$$Y_t = wH_t^Y \tag{2}$$

where H^Y denotes the part of the total stock of human capital that is employed in final goods production and w is a positive constant. The final good sector is characterized by competitive firms who earn zero profit and pay a constant wage rate per unit of skill, given by w (which is the marginal as well as average product of human capital in this AK-technology set up).

2.4 Human Capital Formation

Human capital is acquired through compulsory schooling in public schools. We postulate that human capital acquired by a child through formal schooling depends on two broad sets of factors: (a) the overall quality of schooling, and (b) the absorbtive capacity of the student.

The overall quality of schooling is determined by various inputs provided by the teachers as well as school infrastructure. We focus on two specific teacher-related inputs: (i) how much person attention a student gets from the teacher, and (ii) the average quality of teaching. The former is captured by the teacher-student ratio (θ_t), while the latter is proxied by the average skill level of teachers (h_t^{TA}). These two teacher-related inputs determine the overall quality of schooling through the following education technology[3]:

$$Q_t = (\theta_t)^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}; 0 < \alpha < 1. \tag{3}$$

proportion of agents' income is taxed away to finance the education bill of the next generation. Thus the contribution made towards next generation's education is only a proxy for the educational bequest. The actual amount spent on her own progeny (i.e., per child education expenditure) does not necessarily equal the amount of contribution made by the parent.

[3]Recall that total student population (consisting of the entire young generation) is of measure 1. Thus θ measures the proportion of adult population that is engaged in teaching profession as well as the corresponding teacher-student ratio.

For any given quality of schooling, the skill level acquired by a student also depends on her absorptive capacity. Consider an young agent with inherent ability x who has an absorptive capacity of A_x. Then the skill level acquired by this young agent (to be employed in next period) is given by:

$$h_{t+1}^{x} = A_x Q_t^{\gamma} \; ; 0 < \gamma < 1. \tag{4}$$

The absobtive capacity of a student of course depends on her innate ability. But innate ability can be complemented by hard work. Accordingly, the abstorbtive capacity of a student of ability x, who puts in an effort level e, is given by:

$$A_x = e^{\beta} x^{1-\beta}; 0 < \beta < 1. \tag{5}$$

There are several features of the education and skill formation technology that require further elucidation. First, notice that the education technology that determines the quality of schooling (as specified by equation (3)) is identical to Tamura (2001). However our specification of the human capital formation technology is different from Tamura in that it completely ignores the role of 'home education' in influencing the level of human capital acquired by the next generation. In other words, we do not consider the impact of parental human capital on that of the children. This is **not** to deny the role of parents in the learning process of a child. While the quality of education is certainly influenced by non-school factors such as parental education, home environment etc., in this paper our focus is exclusively on school-specific factors asociated with the formal learning process. Therefore we deliberately shut off other mechanisms of human capital transmission across generations.[4]

Secondly, human capital formation exhibits diminishing returns with respect to individual effort level: greater effort increases the level human capital acquired, but at a decreasing rate ($\beta < 1$). Moreover, inherent ability and effort level are complementary[5]: a person who has higher innate ability *ceteris paribus* also has higher incentive to exert more effort. However whether a more able person actually exerts greater effort in equilibrium will depend upon the wage incentive. This issue will be taken up in detail later in the context of different education policies and concommitant teachers' salary schemes.

[4] There seems to exist some empirical evidence that supports this assumption. For example, Card and Krueger (1992) write: "Controlling for measures of school quality, however, we find no evidence that returns to education are related to the income or schooling levels of the parents' generation." However, there are other empirical studies which have re-iterated the importance of 'home education' (e.g., Woessmann, 2003).

[5] The second order cross partial is positive.

2.5 Individual Choices

Recall that agents derive positive utility form own consumption and the contribution made towards public education and derive negative utility from effort exerted in acquiring education (refer to the utility function specified by (1)). Now under the public education regime a part of the agent's adulthood income is taxed away at a predetermined rate to provide for the education of the next generation. Thus the optimal consumption and bequest choices are trivial: the agent simply contributes the stipulated taxed amount towards educational bequest and consumes her entire after-tax income.[6] Consider a young agent of generation t who is born with an innate ability x and exerts an effort e_t during childhood that generates an adulthood income of $y^x_{t+1} \equiv wh^x_{t+1}$, where $h^x_{t+1} = \left(e_t^{\beta} x^{1-\beta}\right) Q_t^{\gamma}$ (by (4) and (5)). Let τ be the rate at which her income is taxed to pay for the education of the next generation. Substituting for $c_{t+1} = (1-\tau)y^x_{t+1}$, and $b_{t+1} = \tau y^x_{t+1}$, we get the indirect utility of the agent defined in terms of her effort level as:

$$\hat{U} \equiv \tau^{1-\epsilon}(1-\tau)^{\epsilon} w \left(e_t^{\beta} x^{1-\beta}\right) Q_t^{\gamma} - e_t. \tag{6}$$

A forward-looking ratinal agent, who can perfectly ancipate her income,[7] maximizes $\hat{U}$ to choose his optimal effort level at childhood such that

$$\tau^{1-\epsilon}(1-\tau)^{\epsilon} w \beta Q_t^{\gamma} \left(e_t\right)^{\beta-1} x^{1-\beta} - 1 = 0.$$

It is easy to verify that an interior solution exists (since β is a positive fraction). Thus the optimal effort choice of an agent of generation t who is born with an innate ability x is given by

$$e_t^x = \left(\beta \tau^{1-\epsilon}(1-\tau)^{\epsilon} w Q_t^{\gamma}\right)^{1/(1-\beta)} x. \tag{7}$$

The corresponding level of human capital acquired by this agent is

$$\begin{aligned} h^x_{t+1} &= \left[\left(\beta \tau^{1-\epsilon}(1-\tau)^{\epsilon} w Q_t^{\gamma}\right)^{1/(1-\beta)} x\right]^{\beta} x^{1-\beta} Q_t^{\gamma} \\ &= \left[\beta \tau^{1-\epsilon}(1-\tau)^{\epsilon} w\right]^{\beta/(1-\beta)} Q_t^{\gamma/(1-\beta)} x. \end{aligned} \tag{8}$$

Notice that in the current schema of things when each agent' income is proportional to her qualification (level of human capital), the optimal effort level of the agent is proprtional to her ability.

[6] Instead of allowing for a warm glow bequest term (b), we could have defined the utility of an agent as a function of the human capital acquired by her child (e.g. Tamura, 2001), or the quality of education received by her child (e.g. Glomm and Ravikumar, 1992). Since both these varibles are determined by factors which are not directly under the control of the agent, the qualitative analysis would have been the same. Assuming a 'warm glow' educational bequest only simplifies the exposition.

[7] There is no uncertainty in this model. An agent precisely knows his ability index upon birth.

3 Education Policy

Compulsory schooling is provided to every child free of cost and the salary bill of the teachers is financed by the government. We assume that the government offers each teacher her opportunity wage, i.e., the wage that she would be able to earn by employing her skills elsewhere. This implies that if the government wants to employ an agent with human capital h, then it has to pay a salary of wh. (We shall assume that whenever the government pays the market-equivalent salary to any person, that person is willing to engage in teaching, even though wage-wise she is actually indifferent). And the government pays the salary bill of the teachers by taxing the income of the entire working population (current adults) at a constant rate while maintaining a balanced budget.[8]

The objective of the education policy is to allocate the government tax revenue efficiently so as to maximise the overall quality of schooling, Q_t (as in Tamura (2001)). But unlike Tamura (2001) or Gilpin and Kaganovich (2009), we do not impose any minimum qualification (i.e., minimum human capital requirement) to be appointed as teacher. This is because here we are talking about a generic education technology where some degree of schooling can be provided by any agent irrespective of her own level of education. What quality of teachers are to be employed is itself a policy decision. Therefore we do not see the need for imposing an additional exogenous constraint on the quality of teachers.[9]

In maximising the overall schooling quality, the binding constraint faced by the gorvernment is of course its revenue cosnstraint - as represented by the balanced budget condition. But its choices are also restricted by the distribution of human capital in the economy in the following way. For any given teacher-student ratio (θ), the best quality teachers in the economy consists of θ measure of people who are at the very upper end of the distribution. This generates an upper bound on the possible average quality of teachers. Likewise, the worst quality teachers in the economy consists of θ measure of people who are at the very lower end of the distribution. This generates an lower bound on the possible average quality of teachers. We elaborate below the exact specification of all these constraints.

[8] By employing a set of the current adults as teachers, the government shifts a part of the working population from final goods production to teaching. Since salaries of these people have to be paid in terms of the final good, effectively the government has to tax away a part of the final output produed by the rest to pay for the salary bill of the teachers. However, the post-tax wage rates in the teaching and non-teaching (final goods production) sectors being equal, this is equivalent to taxing the income of the entire working population.

[9] In section 6 we consider a different education policy where the government fixes the salary as well as the minimum educational qualification of a teacher. We get contrasting results in the two cases.

Let us assume that the government imposes a proportional labour income tax at a time-invariant rate τ which is fixed arbitrarily at this point.[10] This generates a total tax revenue at time t, given by $T_t \equiv \tau \int_0^1 w h_t^x dx = \tau w H_t$, where H_t denotes the aggregate stock of human capital time t (defined as the sum-total of all adult agents' human capital who differ in terms of ability, i.e., $H_t \equiv \int_0^1 h_t^x dx$). On the other hand the total salary bill of the teachers employed by the government is given by wH_t^T,where H_t^T denotes the aggregate human capital of the teachers. Thus at any point of time t, the balanced budget condition of the government implies the following relationship between H_t and H_t^T:

$$\begin{aligned} \tau w H_t &= w H_t^T \\ &\Rightarrow \; H_t^T = \tau H_t. \end{aligned} \tag{9}$$

Notice that the balanced budget condition itself captures the implied trade-off between teacher quality and teacher quantity. For any given tax rate τ and for any historically given stock of human capital H_t, the RHS of the above equation is fixed. The LHS on the other hand can be mechanically written as $H_t^T = \theta_t h_t^{TA}$. Substituting this in the above balanced-budget condition, one can immediately see the quality-quantity tradeoff involved here:

$$\theta_t h_t^{TA} = \tau H_t. \tag{10}$$

The precise values of θ_t and h_t^{TA} are to be determined by the education policy of the government. However, the RHS of equation (10) being a constant (for a given τ), the equation is represented by a rectangular hyperbola. Thus if the government opts for a higher average quality of teachers then it must compromise in terms of the number of teachers.

The distribution of human capital at any point of time t imposes two additional constraints on the choice of θ_t and h_t^{TA}. First, note that for any given value of θ_t, $H_t^T \leqq \int_{1-\theta}^1 h_t^x dx$, where the RHS represents the aggregate human capital of the upper θ-proportion of the population. Noting that $H_t^T = \theta_t h_t^{TA}$, we can write this condition as follows:

[10] Eventually we shall allow for the optimal choice of τ.However that even when τ is chosen optimally, we will restrict our analysis to time-inavriant values of τ only. The reason for maintaining a time-invariant tax rate is two-fold: (a) we want to focus on a balanced growth path which cannot be attained unless the tax rate is constant over; and (b) we want to keep the incentive structure same for different generations which implies retaining the same policy parameters across generations.

$$
\begin{aligned}
h_t^{TA} &\leq \frac{1}{\theta_t}\int_{1-\theta}^{1} h_t^x dx \\
&= \frac{1}{\theta_t} H_t \frac{\int_{1-\theta}^{1} h_t^x dx}{\int_0^1 h_t^x dx} \\
&= \frac{1}{\theta_t} H_t \frac{\int_{1-\theta}^{1} x dx}{\int_0^1 x dx} \\
&= H_t \frac{1-(1-\theta_t)^2}{\theta_t} \\
&= (2-\theta_t)\, H_t. \qquad (11)
\end{aligned}
$$

Similarly, for any given value of θ_t, $H_t^T \geqq \int_0^\theta h_t^x dx$, where the RHS represents the aggregate human capital of the bottom θ-proportion of the population. Once again, we can write this condition as follows:

$$
\begin{aligned}
h_t^{TA} &\geq \frac{1}{\theta_t}\int_{0}^{\theta} h_t^x dx \\
&= \frac{1}{\theta_t} H_t \frac{\int_{0}^{\theta} h_t^x dx}{\int_0^1 h_t^x dx} \\
&= \frac{1}{\theta_t} H_t \frac{\int_{1-\theta}^{1} x dx}{\int_0^1 x dx} \\
&= H_t \frac{(\theta_t)^2}{\theta_t} \\
&= \theta_t H_t. \qquad (12)
\end{aligned}
$$

In view of the balanced budget condition and the two distributional constraints (equations (10), (11) and (12) respectively), the optimal education policy of the government consists of the solution to the following constrained optimization exercise:

$$
Max_{\{\theta_t, h_t^{TA}\}} Q_t = (\theta_t)^{\alpha} \left(h_t^{TA}\right)^{1-\alpha}
$$

subject to

$$
\begin{aligned}
\text{(i) } h_t^{TA} &= \frac{\tau H_t}{\theta_t}; \\
\text{(ii) } h_t^{TA} &\leqq (2-\theta_t)\, H_t; \\
\text{(iii) } h_t^{TA} &\geqq \theta_t H_t.
\end{aligned}
$$

In solving the above problem, the government takes H_t, the distribution of H_t and τ as given. Figure 1 characterizes the constraint space for this optimization problem.

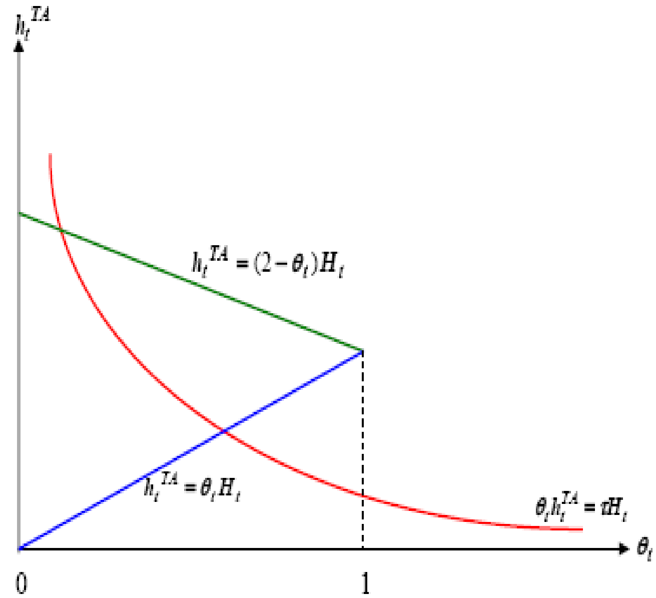

Figure 1

3.1 Optimal Solutions for θ and h_t^{TA}

Since the balanced budget condition (constraint (i)) is always binding, we can use this condition to eliminate one of the choice variables from the objective function. Eliminating h_t^{TA}, we get the reduced-form objective function as: $Max_{\{\theta_t\}}\left[(\theta_t)^{2\alpha-1}(\tau H_t)^{1-\alpha}\right]$. From this reduced-form expression it is obvious that depending on the parameter value α, it is optimal for the government to go either for the maximum possible teacher quantity (whenever $\alpha > \frac{1}{2}$) or for the best possible teacher quality (whenever $\alpha < \frac{1}{2}$).[11] In terms of Figure 1, the optimal policy choice thus involves moving along the $\theta_t h_t^{TA} = \tau H_t$ curve (depicted by the rectangular hyperbola drawn in bold) until one of the distributional constraints binds. More specifically, if $\alpha > \frac{1}{2}$, then it is optimal to move downward along the $\theta_t h_t^{TA} = \tau H_t$ curve until one hits the straight line representing $h_t^{TA} = \theta_t H_t$. On the other hand, if $\alpha < \frac{1}{2}$, then it is optimal to move upward along the $\theta_t h_t^{TA} = \tau H_t$ curve until one hits the straight line representing $h_t^{TA} = (2-\theta_t)\,H_t$. Formally, the optimal solutions for θ_t and corresponding h_t^{TA} for various parametric conditions are described below[12]:

[11] When α is exactly equal to $\frac{1}{2}$, the teacher quality-quantity choice becomes irrelevant and the overall quality of schooling depends only on the aggreagte expenditure on schooling. We ignore this case.

[12] These solutions are obtained by solving a set of simultaneous equations represented by

Case A: $\alpha > 1/2$

$$\theta^* = \sqrt{\tau} \ ; \tag{13}$$

$$\left(h_t^{TA}\right)^* = H_t\sqrt{\tau}. \tag{14}$$

Case B: $\alpha < 1/2$

$$\theta^* = 1 - \sqrt{1-\tau} \ ; \tag{15}$$

$$\left(h_t^{TA}\right)^* = H_t \frac{\tau}{1-\sqrt{1-\tau}}. \tag{16}$$

The corresponding optimal quality of schooling in the two cases are given repectively by:

$$Q_t^* = \begin{cases} \sqrt{\tau}\ \left(H_t\right)^{1-\alpha} & \text{for } \alpha > 1/2; \\ \\ \dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}} \left(H_t\right)^{1-\alpha} & \text{for } \alpha < 1/2. \end{cases} \tag{17}$$

Notice that the optimal schooling quality depends on the current available stock of human capital (H_t) as well as the time-invariant tax rate τ. However the exact functional relationship between Q^* and τ differs depending on whether α is greater or less than 1/2. In particular, for any given stock of aggregate human capital, optimal schooling quality is increasing in τ if $\alpha > 1/2$, but the result is ambiguous if $\alpha < 1/2$. In fact one can easyly verify that when $\alpha < 1/2$, optimal schooling quality Q_t^* increases until $\tau = 4\alpha(1-\alpha)$ and decreases thereafter.[13] The reason for this non-monotic relationship between Q^* and τ is quite simple. If $\alpha < 1/2$, then teachers' average human capital is a more significant input than the number of teachers in the education technology. So it is always optimal for the government to employ the best quality teachers, subject to its budget constraint and subject to the distribution of human capital. In terms of Figure 1, this optimal choice is represented by the point of intersection of the $\theta_t h_t^{TA} = \tau H_t$ curve with the $h_t^{TA} = (2-\theta_t) H_t$ line. Now

(1) $\theta_t h_t^{TA} = \tau H_t$; (2) $h_t^{TA} = \theta_t H_t$ for the case $\alpha > 1/2$;

and by

(1′) $\theta_t h_t^{TA} = \tau H_t$ and (2′) $h_t^{TA} = (2-\theta_t)H_t$ for the case $\alpha < 1/2$.

Also note that the only admissible solutions for θ are those which lie within the interval $[0,1]$.

[13] Taking the partial derivative of Q_t^* with respect to τ, after simplification, one can show that $\frac{\partial (Q_t)^*}{\partial \tau} \gtreqless 0$ according as $\tau^2 - 4\alpha(1-\alpha)\tau \lesseqgtr 0$,where $4\alpha(1-\alpha)$ is a positive fraction (since $\alpha < 1/2$). Thus for any τ such that $0 < \tau < 4\alpha(1-\alpha)$, optimal schoolong quality is increasing in τ, while for any τ such that $4\alpha(1-\alpha) < \tau \leqq 1$, optimal schooling quality is decreasing in τ.

if available resources go up while the stock of human capital remains unchanged, this entails a rightward shift in the $\theta_t h_t^{TA} = \tau H_t$ curve while positions of the two distributional constraints remain the same. Thus the new optimal (h, θ) configuration now shifts down along the $h_t^{TA} = (2 - \theta_t) H_t$ line, which implies that at this new optima, average teacher quality is lower while number of teachers is higher. This is so because the government had already employed the poeple who were at the top end of the distribution. Now as it attempts to employ more teachers (due to a relaxation of the budget constarint), it has to move to a lower quality pool, which necessarily lowers the average quality of teachers. At the same time number of teachers goes up. To a certain extent more teachers compensate for the fall in the teacher average quality. But α being less than 1/2 , teacher quantity is a poor substitute for teacher quality and eventually this shows up in terms of a declining schooling quality.

4 Human Capital Dynamics & Growth

Before we get down to the precise dynamic equation for human capital formation and the associated growth path of the economy, recall that the term H_t^T / H_t denotes the proportion of the aggregate human capital stock that is employed in the teaching professon. Equation (9) tells us that this proportion is directly measured by the tax rate τ. Now output in this economy depends linearly on the part of aggregate stock of human capital that is employed in final good production (H_t^Y). Since τ proportion of the total human capital stock H_t is employed in the teaching professon, it follows that $H_t^Y = H_t - H_t^T = (1 - \tau) H_t$, i.e., H_t^Y is also proportional to H_t. This tells us that the rate of growth of output in this economy can be measured by the rate of growth of the aggregate stock of human capital. In other words,

$$\frac{Y_{t+1}}{Y_t} - 1 \equiv g_t = \frac{H_{t+1}}{H_t} - 1. \tag{18}$$

Now aggregate human capital stock in the next period, $H_{t+1} \equiv \int_a^{a+1} h_{t+1}^x dx$, is determined by the governement's education policy (h_t^{TA} and θ_t) as well as by the effort spent today by each agent in acquiring education (e_t) during childhood. We have already solved for the optimal effort choice by a rational forward-looking agent with ability x and the corresponding level of human capital acquired by her (refer to equations (**??**) and (8) repectively). Thus substituing for h_{t+1}^x and aggregating over all agents who differ in terms of innate abilities, next period's human capital is given by:

$$\begin{aligned} H_{t+1} &= \left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon}w\right]^{\beta/(1-\beta)} Q_t^{\gamma/(1-\beta)} \int_0^1 x dx \\ &= \left[\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon}w\right]^{\beta/(1-\beta)} Q_t^{\gamma/(1-\beta)}. \end{aligned} \tag{19}$$

Using equation (17) to substitute for Q_t , we get the following equation determining the dynamics of the aggregate human capital in this economy:

$$H_{t+1} = \begin{cases} [\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon}w]^{\beta/(1-\beta)} [\sqrt{\tau}\ ]^{\gamma/(1-\beta)} (H_t)^{(1-\alpha)\gamma/(1-\beta)} & \text{for } \alpha > 1/2 \ ; \\ [\beta\tau^{1-\epsilon}(1-\tau)^{\epsilon}w]^{\beta/(1-\beta)} \left[\dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}\right]^{\gamma/(1-\beta)} (H_t)^{(1-\alpha)\gamma/(1-\beta)} & \text{for } \alpha < 1/2. \end{cases} \tag{20}$$

Notice that for any given time-invariant tax rate τ, the economy will exhibit a balanced growth path if and only if

$$(1-\alpha)\gamma = (1-\beta)\,. \tag{ASSUMPTION 1}$$

Henceforth we shall assume that this parametric condition always holds. The corresponding balanced growth rate for Case (A) and Case (B) are given below:

$$g = \begin{cases} (\beta w)^{\beta/(1-\beta)} [\tau^{1-\epsilon}(1-\tau)^{\epsilon}]^{\beta/(1-\beta)} [\sqrt{\tau}\ ]^{1/(1-\alpha)} - 1 & \text{for } \alpha > 1/2 \ ; \\ (\beta w)^{\beta/(1-\beta)} [\tau^{1-\epsilon}(1-\tau)^{\epsilon}]^{\beta/(1-\beta)} \left[\dfrac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}\right]^{1/(1-\alpha)} - 1 & \text{for } \alpha < 1/2. \end{cases} . \tag{21}$$

Since the human capital stock in this economy is growing at a constant rate (given (ASSUMPTION 1)), it is easy to verify (from equations (14), (16), (13), (15) and (17)) that along the balanced growth path (for both Case A and B), optimal teacher-student ratio remains constant but teacher quality increases over time and so does the overall quality of schooling. Proposition 1 below summarises the key characteristics of the optimal education policy under various parametric conditions, and Proposition 2 states the implications of these for the balanced growth path.

Proposition 1 *Consider any given time-invariant tax rate τ. If the education technology is such that teacher-quantity is more important that teacher-quality (i.e., $\alpha > 1/2$) then* *the optimal education policy for the government consists of maintaining a constant teacher-student ratio at the level* $\sqrt{\tau}$ *and employing the least qualified (and lowest ability) agents as teachers in every period. On the other hand if the education technology is such that teacher-quality is more important that teacher-quantity (i.e.,* $\alpha < 1/2$*) then the optimal education policy for the government consists of maintaining a constant teacher-student ratio at the level* $1 - \sqrt{1-\tau}$ *and employ the most qualified (and highest ability) agents as teachers in every period.*

Proposition 2 *Suppose ASSUMPTION 1 holds. Then for any given time-invariant tax rate, the economy grows at a constant rate. Along the balanced growth path, the relative quality of teachers (as characterized by their innate abilities) remains constant while the absolute teacher-quality as well as overall schooling quality increase over time.*

5 Taxation Policy: Optimal Tax Rate

So far we have assumed that the tax rate τ is fixed arbitrarily. However, as is evident from (17), the optimal schooling quality depends crucially on the tax rate chosen by the government. Moreover the choice of tax rate directly affects the growth rate of the economy (see equation (21)). Thus the tax rate itself is another policy instrument which the government can use to influence the educational outcome and the growth of the economy. One cannot talk of the optimal education policy independent of the optimal taxation policy. In this section we discuss the choice of optimal tax rate by the government.

In analysing the optimal choice of tax rate, we consider two alternative regimes. In the first regime the government itself chooses the tax rate with the objective of maximizing of growth. In the second regime the tax rate is decided by majority voting: in each period the government asks the current adults to vote for their most-preferred tax rate and implements the one which is preferred by majority.[14] As we already know from the literature on public expenditure and growth (e.g., Barro (1990); Alesina-Rodrik (1994)), the growth maximising policy need not always co-incide with the majority preferred policy. Here we re-examine this issue in the context of public education.

[14] For the second regime to generate meaningful results (which are comparable to the first regime) two conditions need to be satisfied: (i) there exists a unique majority voting equilibrium tax rate; and (ii) this tax rate is time-invariant. As we shall see, both these conditions are fulfilled in the current set up.

5.1 Growth Maximizing Tax Rate

Suppose the objective of the government is to maximise growth rate of output. Equation (21) gives us the two balanced growth rates under the two mutually exclusive cases. Let us analyse these two cases separately.

5.1.1 Case A: $\alpha > 1/2$

Maximising the relevant balanced growth rate with respect to τ, and using ASSUMPTION 1 , from the first order condition we get:

$$(\beta w)^{\beta/(1-\beta)} \left(\tau\right)^{[\gamma+2\beta(1-\epsilon)]/2(1-\beta)} (1-\tau)^{\epsilon\beta/(1-\beta)} \left[\frac{\gamma+2\beta(1-\epsilon)}{2(1-\beta)}\left(\frac{1}{\tau}\right) - \frac{\epsilon\beta}{(1-\beta)}\left(\frac{1}{1-\tau}\right)\right] = 0$$

The resulting optimal tax rate is given by:

$$\tau^{*} = 1 - \left(\frac{2\beta}{\gamma+2\beta}\right)\epsilon. \tag{22}$$

This optimal tax rate arises here due the following trade-off. We have already seen that next period's human capital depends on the effort exerted by the students as well as the teaching quality (refer to equation (19)). Optimal teaching quality in the case is a monotonically increasing function of τ. But optimal effort choice depends non-monotonically on τ. On the one hand, a higher tax rate means that a lower proportion of the agents' income will be available to them for consumption tomorrow. This reduces their incentive to exert effort. On the other hand, a higher tax rate also means that they will effectively leave more educational bequests for their progeny. To the extent agents care for their progeny (due to presence of 'warm glow'), this gives them some satisfaction and therefore increases their incentive to exert effort. This trade-off in effort choice results in the existence of a growth maximising tax rate (after adjusting for the positive influence of improvement in schooling quality - which somewhat mitigates the effect of effort-related trade-off but cannot eliminate it completely.).

5.1.2 Case B: $\alpha < 1/2$

The derivation in this case is more involved for the simple reason that now there are two trade-offs associated with an increase in τ : one is the trade-off related to effort choice; other is the trade-off related to schooling quality. These two trade-offs are captured by the terms $[\tau^{1-\epsilon}(1-\tau)^{\epsilon}]^{\beta/(1-\beta)}$ and $\left[\frac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}\right]^{1/(1-\alpha)}$ respectively in the expression for the balanced growth rate (refer to equation (21)). Let us define $F(\tau) \equiv \tau^{1-\epsilon}(1-\tau)^{\epsilon}$ and

$G(\tau) \equiv \frac{(\tau)^{1-\alpha}}{\left(1-\sqrt{1-\tau}\right)^{1-2\alpha}}$. Since both $F(\tau)$ and $G(\tau)$ enter with postive powers in the growth rate expression, what happens to the growth rate depends crucially on what happens to these two functions as τ increases. It is easy to verify that $F(\tau)$ is maximised at $\tau = 1-\epsilon$, while we have already seen that $G(\tau)$ (which is nothing but the optimal schooling quality) is maximized at $\tau = 4\alpha(1-\alpha)$ (see footnote 13). Thus it is obvious that the balanced growth rate will be increasing in τ for any $\tau \leqq Min\,[1-\epsilon, 4\alpha(1-\alpha)]$; will be decreasing in τ for any $\tau \geqq Max\,[1-\epsilon, 4\alpha(1-\alpha)]$; and will attain a maxima somewhere in between. In other words, there exits an optimal tax rate τ^* such that $1-\epsilon < \tau^* < 4\alpha(1-\alpha)$ if $(1-\epsilon) < 4\alpha(1-\alpha)$; and $4\alpha(1-\alpha) < \tau^* < 1-\epsilon$ if $4\alpha(1-\alpha) < (1-\epsilon)$.

5.2 Majority-preferred Tax Rate

Suppose now that the government allows the tax rate to be chosen by majority voting. Thus in each period the government chooses a tax rate τ_t on the basis of the preferences of the currently adult agents. Notice that in our formulation, this tax rate τ_t enters into the utility function of the adults in two ways. The current tax rate enters directly through the disposable income $(1-\tau_t)y_t$ which goes into her consumption and through the bequest $\tau_t y_t$ which goes into the education of her child. But the (anticipated) tax rate τ_t had also influneced her optimal effort choice in the previous period: e_{t-1}. Thus is they are asked to choose that tax rate today, they choose the current tax rate so as to maximise their indirect utility:

$$\hat{U}^x = (1-\tau_t)^{\epsilon}\,(\tau_t)^{1-\epsilon}\,y^x(e^*(\tau_t)) - e^*(\tau_t).$$

By Envelope theorem, this indirect utility is maximized at

$$\tau^{**} = 1-\epsilon. \tag{23}$$

There are several important implications of this result. First, note that the tax rate chosen by an agent is independent of x. In other words, all agents, irrespective of their innate ability will choose the same tax rate τ^{**}, which will obviously be the majority-preferred tax rate. Secondly, the majority-preferred tax rate in indeed time-invariant: in every period the current adults choose the same tax rate $1-\epsilon$. Thirdly, a comparison of the majority-preferred tax rate with the growth maximising tax rate clearly reveals that as long as schooling quality is increasing in τ, $\tau^{**} < \tau^*$, i.e., the growth maximizing tax rate is always higher than the majority-preferred tax rate.[15] This result is due to the fact the in choosing their most-

[15] This happens if either $\alpha > 1/2$, or $\alpha < 1/2$ and $(1-\epsilon) > 4\alpha(1-\alpha)$.

preferred tax rate, the agents only look at their consumption-bequest trade-off (which has an impact on their optimal effort choice as well). But if the tax rate is time-invariant, then it also has an indirect positive effect on the income of the current adults through the quality of schooling factor. But the current adults do not care about this latter effect since it operates though a tax that was paid by their parents. Thus the 'myopic' adults will underinvest in children's education and the economy will attain a growth rate which is lower than the maximum. Proposition 3 below summarises these results.

Proposition 3 *Consider an economy which is on a balanced growth path. There exists a unique tax rate that maximizes the rate of growth output in this economy. There also exists a time-invariant unique tax rate that is preferred by majority.*

6 Conclusion

In this paper we have analyzed the impact of overall quality of education on growth in a framework where better quality of schooling results from the presence of better quality teachers as well as presence of higher number of teachers (reletive to the class size). We show that overall quality of schooling is an important factor contributing to growth. However in an economy where education is publicly provided and is financed by taxation, the requirement of maintaining a balanced budget on the part of the government imposes a trade off in terms of quality and quantity of teachers. For any given amount of tax revenue, in order to attract better quality individuals to teaching profession requires paying a higher salary bill, which in term implies that the less number of teacher can be employed. Since the quality and quantity of teachers enter differently in the education technology, the government has to optimally allocate of its limited resources between teacher quality and teacher quantity in order to ensure efficiency.

Apart from the implied trade off between teacher quality and teacher quantity (which is to be resolved by an efficient education policy), there is another trade off implicit here. This second trade off arises in terms of the taxation policy of the government. Higher taxation eases the balancec budget constraint of the government, which enable it to improve the overall quality of schooling. This has a positive impact on the human capital formation process and therefore on growth. On the other hand, there is a negative impact of higher taxation on the incentive to invest effort in skill formation, which undermines the impetus to growth. This trade off between quality of schooling and effort exerted by the students generates a hump-shaped relationship between tax rate and growth. Since effort has a negative utility cost, this also explains why the growth maximizing tax rate cannot be welfare-maximizing. The welfare maximising tax rate is always lower that the growth maximizing one.

In the context of the trade off between schooling quality and students' effort, it is important to note that in our model we have assumed that individual ability is observable and is known to an agent when she decides how much effort to invest in acquiring human capital. This assumption however is not crucial for our results. One can easily show that the same conclusion will prevail even when agents do not know their exact ability but optimize on the basis of the expected value of ability. In this latter case, the optimal effort chosen will be the same across all agents; however the actual level of human capital will differ across agents depending on their actual levels of ability. Since the nature of the quality-effort trade off in individual skill formation is the same irrespective of the assumed level of ability, the basic results of the model will remain unchanged. A more interesting scenario arises when the human capital formation technology depends on the teachers' average ability rather than average human capital. If human capital is observable but innave ability is not, then employing teachers solely on the basis of level of human capital acquired may give rise to adverse selection hampering growth. Full exploration of this idea however lies beyond the scope of the present paper.

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