Aggregate Fertility and Household Savings: A General Equilibrium Analysis using Micro Data*

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March 28, 2014

Abstract

This study uses micro data and an OLG model to show that general equilibrium forces are critical for understanding the relationship between aggregate fertility and household savings. First, we document that parents perceive children as an important source of old-age support and that in partial equilibrium, increased fertility lowers household savings. Then, we construct an OLG model that parametrically matches the partial equilibrium empirical evidence. Finally, we extend the model to conduct a general equilibrium analysis and show that under standard assumptions and with the parameters implied by the data, general equilibrium forces can substantially offset the partial equilibrium effects. Thus, focusing only on partial equilibrium effects can substantially overstate the effect of a change in aggregate fertility on households savings.

Keywords: Savings, Demographic Structure, Fertility

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*We are grateful to Mark Aguiar, Paco Buera, Nicolas Ceourdacier, Oded Galor, Mikhail Golosov, Hui He, Jonathan Heathcote, Nobu Kiyotaki, Samuel Kortum, David Lagakos, Costas Meghir, Benjamin Moll, B. Ravikumar, Richard Rogerson, Mark Rosenzweig, Ananth Seshadri, Todd Schoelman, Yong Shin, Michael (Zheng) Song, Kjetil Storesletten, Gustavo Ventura, Shangjin Wei, David Weil and Fabrizio Zilibotti for their insights; the participants at the Yale Development Lunch, Yale Macro Lunch, EIEF Lunch Seminar, the conference on Human Capital and Development at the Washington University of St. Louis, the NBER Summer Institute Income Distribution and Macroeconomics, Conference of "China and the West 1950 – 2050: Economic Growth, Demographic Transition and Pensions" in Zurich, the Symposium on China's Financial Markets, and the NBER China Workshop for comments; and Yu Liu and Xiaoxue Zhao for excellent research assistance.

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1 Introduction

The linkage between aggregate fertility change and economic performance is central to models of economic growth. A large literature has provided important evidence relating aggregate fertility change to growth (e.g., Romer, 1986; Kremer, 1993; Jones, 1999; Galor and Weil, 2000), growth and inequality (e.g., De La Croix and Doepke, 2003), culture (e.g., Fernandez and Fogli, 2006; Fernandez and Fogli, 2009), social security (e.g., Boldrin and Jones, 2002; Boldrin, De Nardi, and Jones, 2005; Song, Storesletten, Wang, and Zilibotti, 2012) and savings (e.g., Becker and Barro, 1988; Barro and Becker, 1989; Manuelli and Seshadri, 2009). In particular, Modigliani and Cao (2004) argues that changes in aggregate fertility can also lead to significant changes in household savings through its effect on the dependency ratio and wage growth. They support their claim with descriptive time series data from China, where a substantial reduction in fertility during the 1970s and 1980s, as a result of family planning policies, was accompanied by a rapid rise in savings rate.

Such time series correlations are obviously difficult to interpret, since aggregate fertility change is likely to coincide with other macro economic changes such as changes in the returns to human capital, or relative female wages. In the case of China, one is additionally concerned of the possibility that the increase in savings and the reduction in fertility are both consequences of the massive economic reforms that took place. Moreover, fertility is likely to affect savings through mechanisms other than the pure aggregation channel proposed by Modigliani and Cao (2004). The recent literature has therefore taken advantage of more specific demographic shocks (e.g., the introduction of China's family planning policies, the implementation of family policies in Bangladesh under the leadership of the International Centre for Diarrhea Disease Research, or the birth of twins) to empirically estimate the causal effect of fertility changes on savings. These studies find large negative effects of fertility on savings (e.g., Banerjee, Meng, and Qian, 2011; Choukhmane, Coeurdacier, and Jin, 2013; Ge, Yang, and Zhang, 2012; Ruthbah, 2007).[1] Studies such as Choukhmane, Coeurdacier, and Jin (2013) and Curtis, Lugauer, and Mark (2011) then use the evidence from micro data to calibrate partial equilibrium OLG models to understand the quantitative effect of an aggregate fertility change on savings.

While these studies provide compelling evidence that fertility affects savings decisions, in most cases they cannot give us the correct quantitative effect of a change in aggregate fertility on savings. This is because an aggregate change in fertility has the potential to affect other economic factors that affect savings such as the interest rate and rate of wage growth through its effect on the capital-labor ratio (e.g., Barro and Becker, 1989; Galor and Weil, 1996).[2] The quasi-experimental micro evidence which rely on comparisons of households with different levels of fertility within the same economy will always net out such general equilibrium effects, but we need to take them in to account to get the correct full equilibrium estimate of the impact of a change in fertility. In particular, the fact that higher fertility leads to higher future interest rates and to slower wage growth, both of which may lead to higher savings rates, has the potential to partly undo the negative partial equilibrium effect of fertility on savings that is estimated in the micro empirical analyses.

The goal of this paper is to use a combination of parameter estimates from natural experiments and other micro data and careful modeling to understand whether we need to take these general equilibrium effects seriously in drawing macro policy

[1] This paper supersedes Banerjee, Meng, and Qian (2011).

[2] In their seminal work, Barro and Becker (1988, 1989) model children as consumption and introduction endogenous fertility and intergenerational transfers to optimal growth models. Becker and Barro (1988) uses an open economy framework, where interest rates are exogenous. Barro and Becker (1989) uses a closed economy framework where fertility increases the capital-labor ratio and interest rates. Note that the main difference between our framework and theirs is that we view children as an investment good. This is discussed in detail later in the introduction.

conclusions from micro empirical estimates. While the principle that general equilibrium effects matter is widely accepted (e.g., amongst others, see Heckman, Lochner, and Taber, 1998 and Acemoglu, 2010), concrete examples of their potential quantitative importance are scarce. To the best of our knowledge, we are the first study of the relationship between aggregate fertility and savings to do so. Amongst the broader set of studies related to household savings, there are two that make this methodological point. Weil (1994) notes that aggregate savings is negatively associated with the size of the elderly population despite the lack of micro evidence that the elderly dis-saves. To reconcile these patterns, he theorizes that the elderly saves to make substantial bequests, and the anticipation of income from bequests causes children to save less. More recently, Buera, Kaboski, and Shin's (2012) finds that the redistributive impact of micro finance is stronger in general equilibrium than in partial equilibrium, but the impact on aggregate output and capital is smaller in the latter. Thus, when general equilibrium effects are accounted for, scaling up micro finance programs will have a smaller impact on per-capita income than the implied effect of the partial equilibrium estimates.

Our study proceeds in several steps. First, to motivate the study and obtain parameter values for calibrating the model later in the paper, we use recent survey data to document that parents in China perceive children as their main source of old-age support. At the time of this study, there was no data that contained both total fertility history and data on income and expenditures. Thus, we collected a nationally representative survey to document that the shift in Chinese family planning policies from pro-natal to anti-natal reduced fertility and increased household savings.[3] The empirical findings are consistent with models where children are treated as investment goods on the grounds that they often provide financial and psycholog-

[3] Xin Meng conducted the RUMiC survey in 2008. This is discussed more in the section on data.

ical support to elderly parents (e.g., Caldwell, 1978; Weil, 1997; Boldrin and Jones, 2002).[4]

Next, we characterize the savings decision in a parsimonious Diamond-style OLG model with the additional feature that parents anticipate transfers from children when making savings decisions. We calibrate the partial equilibrium version of this model to match the empirical findings. Then, we introduce general equilibrium effects to our model by endogenizing interest rates (e.g., Barro and Becker, 1989; Galor and Weil, 1996). We find that GE effects can either dampen the partial equilibrium effects of an increase in fertility or exacerbate them (or leave them unchanged). The reason general equilibrium effects may be more muted than the partial equilibrium effect is that the rise in the interest rate and the fall in wage growth reduces the present value of future transfers from children and thus induce parents to save more. The reason for why the GE effect may be stronger has instead to do with the income effect from the rise in interest rates. Therefore, what actually happens will depend crucially on parameter values. Using the parameter estimates we obtain from the micro-empirical analysis, we find that the general equilibrium effect of increased fertility is only 30% of what the partial equilibrium effect estimated from micro data. This is true as long as the inter temporal elasticity of substitution is not too far below one, which seems consistent with the data.

We consider a number of extensions of our model that bring in endogenous fertility, endogenous transfer rates and endogenous human capital investments. Our results are robust to these extensions.

[4]Caldwell (1978) argues that children provide old-age security. Weil (1997) finds that intergenerational transfers occur in both directions – from parents to children and from children to parents. Boldrin and Jones (2002) uses a growth model to formalize the ideas of Caldwell (1978) and show that it can account for demographic patterns in the data. Boldrin, De Nardi, and Jones (2005) goes further to argue that if children provide old-age security, then observed cross-country differences in fertility rates can be observed by cross-countries in social security. Galor (2012) agrees that children provide old-age support to parents, but argues that cross-country differences in social security are quantitatively insufficient for explaining cross-country differences in fertility.

The key contribution of our paper is to provide a concrete example of the importance of general equilibrium effects for underestanding how a shift in aggregate fertility affects savings. Applying partial equilibrium estimates to macro policy without interpreting the results with the appropriate model in this case can be very misleading. At the same time, our study illustrates the importance of obtaining reliable micro evidence since the quantitative effects are highly sensitive to parameter values.

For policy makers in China, our results indicate that abandoning family planning policies and allowing fertility to rise, if our model is to be believed, will have little effect on household savings.

Relative to the literature, our study makes several contributions. First, we address the general methodological concern that there is often a "discordance between the macro models used in policy evaluation and the microeconomic models used to generate the empirical evidence" (Browning, Hansen, and Heckman, 1999). Together with Buera, Kaboski, and Shin (2012), our study aims to be an example of the view that growth models should "build up" from well-identified parameters estimated using experimental and quasi-experimental data (Banerjee and Duflo, 2005).

Second, we add to studies that explore the effects of aggregate fertility change. For example, De La Croix and Doepke (2003) find that endogenous fertility can generate the negative relationship between inequality and growth. In considering quantity-quality tradeoffs in the extension of our model, our paper is related to well-known work of Becker, Murphy, and Tamura (1994), which develops a model that leads to an equilibrium with high fertility and low human capital and an equilibrium with low fertility and high human capital; and Galor and Weil (2000), which develops a unified growth model to describe the historical evolution of population, technology, and output; and Manuelli and Seshadri (2014), which argues that the demographic structure of poor countries both implies less human capital investment per person (due to lower life-expectancy), and to lower aggregate human capital (because young people have less human capital). In emphasizing the macro effects of demographic changes in the contemporary Chinese context, our study is closely related to Song, Storesletten, Wang, and Zilibotti (2012), which shows that the demographic transition in China implies that pay-as-you-go pension systems have redistributive effects across generations.

Finally, our study adds to recent studies that attempt to explain Chinese savings rates that we discussed earlier. We obtain the same negative partial equilibrium effect of fertility and savings as these other recent studies that have used careful empirical strategies to study the effects of fertility and household savings. The key difference is our focus on general equilibrium effects, which has not been mentioned in earlier works. Our work is also related to studies that have explored the role of mechanisms that drive household savings other than fertility. For example, Song and Yang (2010) elaborates Modigliani and Cao's (2004) argument and provides evidence that link the spike in aggregate savings, the growth rate and the flattening of experience profiles over time. Chamon and Prasad (2010) provides evidence that financial underdevelopment and the precautionary motive are important contributors to savings. Similarly, a recent study by He, Huang, Liu, and Zhu (2014) find that precautionary saving and increased employment risk due to the downsizing of the state sector to be important determinants of household savings. Finally, Wei and Zhang (2011) shows that savings rates for middle age parents today are partly driven by the anticipation of paying "bride prices" for sons in a future where there will be many more men than women in the marriage market.

This paper is organized as follows. Section 2 documents that parents believe that children are the main source of old age support. Section 3 documents the relationship between fertility change and savings. Section 4 presents the results from the model, including the calibration of the parameters and the quantitative estimates. Section 5 offers concluding remarks.

2 Children as Old Age Security

Children are arguably seen as one of the most important savings vehicle in China. A typical household has few other instruments for savings. Money can be deposited in banks or credit cooperatives or it be held as cash, but these institutions offer very low interest rates. During the 1980s, annual real interest rates for savings deposit ranged from 0.7 to 1%. In the late 1990s, with the privatization of the urban housing stock, housing became an important savings vehicle. More recently, reforms of financial markets have allowed a small number of urban households to invest in stocks, but despite this, in 2007, almost all household savings (other than housing) in urban areas were in bank deposits (He and Cao, 2007).[5]

It is therefore no surprise that the norm in Chinese society continues to follow the Confucian principle of parents investing in children (or specifically in their sons, and in particular the eldest son) with the expectation that they will be taken care of by their children (again mainly by sons) in old age. Indeed there is a proverb in Chinese that tells parents to "raise children for old age as one would store up grain against famine" (Delehanty, Ginzler, and Pipher, 2008: p. 17).

The *China Health and Retirement Longitudinal Surveys* (CHARLS), which were representative surveys of Chinese households conducted in 2008 and 2011, supports the view that children are important for old age support even today. Several interesting facts emerge. First, the CHARLS asks "Whom do you think you can (most) rely on for old-age support?". Around 70% of all respondents, who are 45 years of age or older, reply "children" as the answer, and the choice of answer is uncorrelated

[5] According to the 2002 round of the China Household Income Project (CHIP), average urban households hold approximately 10% of their total savings in stocks and bonds.

with the age of the respondent, which suggests that the norm is not changing very quickly.[6]

There is also limited empirical support from data on cohabitation and transfers. In the data, over fifty percent of elderly parents (over age 65) cohabit with adult children. Adult sons are more than five times more likely to live with elderly parents than adult daughters.[7] These facts are consistent with the belief that children provide support and that sons provide more than daughters to the extent that cohabitation reflects transfers from parents to children.[8] Transfers are only reported for those not cohabiting with their children.[9] For elderly parents (age 65 and older) that do not cohabit with adult children (age 35 and older), the data show that parents with more children have a higher probability of receiving transfers. For example, approximately sixty percent of parents with two or more children receive any transfers, while only twenty percent of parents with fewer than two children receive any transfers. Adult sons transfer twice as much as adult daughters.

Thus, the qualitative and quantitative evidence are consistent with traditional norms of children providing support for elderly parents, the belief that more children result in more support and that sons provide more support than daughters.

[6]These are reported by the 2011 wave. The choice set comprises: “Children”, “Savings”, “Pension or retirement salary”, “Commercial pension insurance” and “Other”.

[7]These are reported by the 2008 Pilot Wave. The 2011 Wave does not yet allow us to identify this information.

[8]A caveat for interpreting cohabitation is that cohabitation may also result from parents providing support to children (e.g., parents subsidize adult children’s housing). However, we find that parents who own their housing are fifteen percent less likely to cohabit with adult children.

[9]These data are only from the 2008 pilot wave because the larger 2008 and 2011 waves do not yet allow the linkage of transfer data. Thus, because of sample constraints, we do not separate urban from rural areas. To the extent that urban households rely less on children and more on employer or state provided pensions, this means that the descriptive statistics we provide overstate the reliance of parents on children in urban areas. Thus, they should be interpreted cautiously as stylized facts that make a qualitative point.

3 The Effects of Fertility on Savings

3.1 Family Planning Policies

The early communist government (1949 -) had a pro-natal stance on fertility (Chang, Lee, McKibben, Poston, and Walther, 2005; Scharping, 2013). Most famously, Ma Yinchu's "New Population Theory", which argued that a rapidly growing population would hinder economic development and that the government should implement population control policies, was officially discredited as being pro-Malthusian and anti-Socialist (Yang, 1986). The government pursued policies that encouraged fertility such as conditioning food rations based on the number of family members and making access to contraceptives difficult until a certain number of children had already been born. Discussions about curbing population growth were confined to the top policy makers until the early 1970s. However in 1971, Mao Zedong and Zhou Enlai made a sudden public policy shift and announced that "population must be controlled", which signaled a turning point in family planning policy practice in China.[10] Efforts began in earnest in 1972. On January 17, 1972, provincial leaders attended a meeting organized by the Ministry of Public Health where the central government demanded that local governments publicize and enforce Mao's instructions on family planning, and instructed all levels of government to establish or reinforce their bureaucracies for organizing or implementing family planning related tasks. In May of that year, the Ministry of Public Health organized a national workshop on family planning measures where all provinces had to participate. These measures stated and clarified the shift in family planning policy and energized the bureaucracy.

[10]On Feb. 15th, 1971, Zhou Enlai re-emphasized the importance of family planning when meeting with the provincial representatives at the National Planning Conference in Beijing: "It's important to control population growth. Government should advocate late marriages and birth control, and vigorously publicize these policies from now on. On July 8th, the State Council published "the Report on Doing Well in Family Planning". The written instruction by the State Council on the document pointed out that "Family planning is an important issue that Chairman Mao has advocated for years. All levels of officially must treat the issue seriously."

By 1973, 23 provinces had established the necessary bureaucracies for implementing family planning related policies.[11]

Our study focuses on the unanticipated initial shift in family planning policy from anti-natal to pro-natal that occurred in 1972, which encouraged birth spacing of three to four years. An unanticipated increase in birth spacing is likely to reduce total fertility since, for example, some mothers will become too old to have a second child after the required waiting period. In urban China, the reduction due to birth spacing was magnified by the subsequent introduction of the One Child Policy in 1980 (1979 in Shanghai), when the government took the unanticipated and unprecedented move of restricting to having only one child.[12] When this occurred, parents who had their first child after 1976 (1975 in Shanghai) and were waiting to pass the required birth spacing to have their second child found that they would remain one child families.

Similar policies were introduced in rural areas, but there was more flexibility across regions and over time.[13] For the sake of simplicity, we only examine urban areas in our analysis.

3.2 Estimating the Effect of Fertility on Savings

We will infer the effect of fertility on savings rates for late-middle aged parents from two reduced form relationships: i) family planning reduced fertility; ii) fam-

[11]The details of family planning policy history public information and documented (in Chinese) by the China Population Information Network (POPIN), a branch of the China Population Development and Research Center (CPDRC or CPIRC). See http://www.cpirc.org.cn/yjwx/yjwx_detail.asp?id=308.

[12]The One Child Policy (OCP) punished households that had more than one child with fines, job loss, and the loss of access to public goods, and rewarded those with only one child with bonuses. Family planning polices also became better defined over time. For example, in 1978, the state defined details on things such as what counted as late marriages and the bonuses and subsidies for workers and farmers if they go through sterilizing operations, etc. See "The Report on the State Councils Family Planning Groups First Meeting" (1978).

[13]The variation in the implementation of the One Child Policy in rural China can be seen in the *China Health and Nutritional Survey*, which reports the relaxations of the policy that are allowed at the community and year level. In contrast, the data show very little variation in these variables across communities or over time for urban areas.

ily planning increased savings. Since parents traditionally rely on sons more than daughters, "fertility" from the perspective of parents thinking about future transfers is some weighted sum of children, where daughters receive less weight than sons. As we do not know these weights, we simply treat daughter and sons separately and estimate the following reduced-form equation

$$y_{ij} = \delta p_{ij} + \alpha m_{ij} + \zeta(p_{ij} \times m_{ij}) + \Delta X_{ij} + \theta_j + \varepsilon_{ij}. \tag{1}$$

y_{ij}, for household i living in region j that had their first child in year t, represents outcomes like the total number of children, savings, etc. We specify that it is a function of: a dummy variable for whether the first child was born after 1972, p_{ij}; a dummy for whether the first child is male, m_{ij}; the interaction term between p_{ij} and m_{ij}a vector of household-level controls, X_{ijt}; region fixed effects, γ_i; and a household-specific error term, ε_{ij}. The standard errors are clustered at the sex (of the first child), year of birth (of the first child) and city level for all of our results.[14] δ is the effect of having a first child in 1972 or afterwards for households that have a daughter for the first child. $\delta + \zeta$ is the effect of having a first child in 1972 or afterwards for households that have a son for the first child.

The hypotheses we are testing are standard given the idea that children, especially the male first child, plays a key role in providing old age support to parents. The claim that having one's first child during or after 1972 decreased total fertility both when the first child is female and when he is male, translates into a test for whether both $\hat{\delta} < 0$ and $\widehat{\delta + \zeta} < 0$. Similarly, the claim that parents rely more on sons than daughters for old-age support, and therefore parents who gave birth after

[14]There are 131 clusters. We can alternatively cluster the standard errors at the sex and year of birth (of the first child) level and then correct for the small number of clusters by estimating wild bootstrapped standard errors. The first stage and reduced form estimates are very similar between these two levels of clustering. There is no correction for the small number of clusters for the 2SLS estimates.

1972 and had a first male child need to save less and can retire earlier compared to parents who gave birth after 1972 and have a first female child would imply, $\hat{\zeta} < 0$ in the savings equations. The vector X_i includes household-specific controls that we will discuss and motivate later as they become relevant.

For a sense of the implied magnitudes, we also estimate an instrumental variables specification, which assumes that the only thing that changed in 1972 for this population was the number of children they could have.

$$y_{ij} = \delta n_{ij} + \alpha m_{ij} + \zeta(n_{ij} \times m_{ij}) + \Delta X_{ij} + \theta_j + \varepsilon_{ij}. \tag{2}$$

Here, n_{ij} is the number of children the family eventually had. The instrumental variables estimates are intended to be illustrative since there are many potential violations of the exclusion restriction. It is possible, for example, that even if the actual number of children were unaffected, the option of having another child later in life might have independent effects.

There are several important facts to keep in mind for our empirical analysis. First, the policies for population control gradually tightened over time. This means that the effect of family planning policies on total fertility is not uniform across households that have their first child after 1972; the later they have their first child, the fewer children they will have. This does not affect the validity of our strategy, but is important for keeping in mind when interpreting the magnitude of the estimates, which give the average post-reform effect.

Second, family planning policy is relatively uniform across urban areas (e.g., Ebenstein, 2010; Qian, 2009) and there are relatively few ethnic minority households (who get some exemption from the policy in most Chinese cities). In any case, variation across cities does not affect the validity of our empirical strategy, which estimates the average change after 1972.

Third, there is little sex selection in our sample. Female infanticide rates in urban China are very low and we restrict our sample to households that bore children before sex-selective abortion became available in the 1980s. Consistent with no sex-selection, 50.3% of all children in our sample are male. Thus, we interpret the coefficient for the sex of the first child, m_{ijt}, as exogenous. Also, note that given the introduction of family planning policies, we have many fewer observations for second or higher parity children than for first parity children and for that reason, our sample size is not large enough for examining the differential effects of male and female higher parity children.

Finally, our identification strategy assumes that the shift to fertility control in the early 1970s was unanticipated. For example, if parents anticipated fertility control policies, those who desired more children may have had more children than otherwise in the years leading up the the policy. This would cause an "Ashenfelter dip" and our strategy will over estimate the effect of the policy on reducing the number of children. If parents that intentionally had more children also had a lower propensity to save for reasons unrelated to fertility, this will also cause our strategy to overestimate the effect of the policy on increasing savings. The historical evidence discussed earlier suggest that it is very unlikely that there was anticipation. To the best of our knowledge, no existing study of family planning in China mentions this possibility.

There are two important caveats to our strategy. First, households in the control group (e.g., those that have their first child prior to 1972) will on average be older than those in the treatment group (e.g., those that have their first child after 1972), which can affect savings patterns if parents of the two groups are at different parts in their life cycle. One way to address this is to control for the age of the household head. However, while this controls for age, it can introduce selection bias if parents choose fertility timing based on factors that are correlated with savings later in life.

This raises a second difficulty. For example, parents that have children later in life may be more risk averse, which will, in turn, cause them to save more. To investigate this possibility, we directly examine the correlation between age at first birth and savings, controlling for the same baseline controls. We find no correlation. We will discuss this in further detail when we interpret the results.

3.3 Data

To document the relationship between fertility and household savings, we use the urban household portion of the larger survey that we collected called the 2008 *Rural-Urban Migration in China* (RUMiC). This is the only data that allows us to measure both the total number of children ever born and savings rate for a sufficient number of households.[15] In this paper, we only use the urban data because family planning policies and access to savings instruments were relatively uniform in urban areas, and equally importantly, because there was little sex-selection. The data is organized as a household-level birth cohort panel according to the birth year of the first child. The empirical analysis focuses on households that had their first child five years before or after the policy shift in 1972, i.e., 1967-77. Almost all households in our sample are married and have at least one child. We end the sample in 1977 because the One Child Policy begins to be binding for households that had their first child around 1977.[16] For symmetry, we begin the sample for parents that had their first child in 1967. Figure 1a shows the kernel density plot for the distribution of the ages of first born children in our sample.

We restrict sample to households headed by individuals who are 50 to 65 years

[15]See the Data Appendix for a detailed discussion of the RUMiC and other survey data from China.

[16]Recall that the One Child Policy was introduced in most cities in 1980. Prior to this, the government followed a less restrictive policy that encouraged parents to space children to be three or four years apart (see Section 3). Thus, parents that had their first child in 1977 could potentially have a second child (the first child would be around three years old), while parents that had their first child after 1977 would have lost the chance of having a second child.

of age to focus on a point in the lifecycle when individuals are most likely to be saving for their retirement. This is the period of the life-cycle when children require relatively little expenditure from parents, when parents are still working, and when children are not yet making transfers to parents. Figure 1b is a kernel density plot for the distribution of the ages of the household heads in our sample. There are very few households with children living at home in our sample.[17] The narrow age band is advantageous because individuals are likely to be on the same part of the life-cycle and therefore comparable to each other. Note that this sample differs from the sample of elderly parents age 65 or older we examined in the Section 2 to document transfer and cohabitation.

The final sample contains 475 households in eighteen cities. Table 1 shows the descriptive statistics. Households in our sample on average have total incomes of 49,584 RMB and expenditures of 32,421 RMB. Savings, the difference between total income (except for transfer income) and total expenditures, are on average 17,162 RMB.[18] The average savings rate in the same is 26%. Figure 1d plots the kernel density of household savings in our sample. It is approximately normally distributed and takes negative as well as positive values. Figure 1e plots the kernel density of household savings rates in our sample.

The average household has approximately two children, 50.3% of which are male. On average, parents had their first child in 1973 and their youngest child in 1976. This means that when the survey was conducted in 2008, households in our sample on average had children age 32-35 year of age. Our sample contains households headed

[17] In our sample, there are only five households with any children under the age eighteen or younger and only fifteen households with any children age 22 or younger. Figure 1c plots the kernel density plot of the distribution of the youngest children in our sample.

[18] These variables are defined in detail in the Data Appendix. In results not presented in this paper, we used several alternative definitions of expenditures, such as with or without including social security contributions (which can be viewed partly as a form of savings). This makes little difference to our results and are not presented for brevity. They are available upon request.

by individuals 51-65 years of age. On average, household heads are approximately 61 years of age and have approximately ten years of education (i.e. one year of high school education) and approximately 42% of our sample is headed by women.[19]

3.4 Results

The Effects of Family Planning on Fertility Table 2 presents the estimated effects of the introduction of family planning on fertility. Column (1) shows a specification that only controls for city fixed effects. The estimates show that parents that gave birth to their first child in 1972 or afterwards had 0.6 less kids on average. This is consistent with the discussion in Section 3. In columns (2) and (3), we add controls that we motivate later when we examine savings. For the examination of fertility, the added controls make little difference. All of the estimates are statistically significant at the 1% level.

In column (4), we estimate equation (1) where we add controls for whether the first child is a son and the interaction of that term with whether the first child was born after 1972. The coefficient for whether a child was born after 1972 reflects the effect on households that have daughters for a first child. The sum of this coefficient and the interaction of whether the first child is a son reflects the effect on households that have a son as a first child. This joint estimate and its p-value are shown at the bottom of the table. The estimate for the uninteracted post-1972 term shows that parents who had their first daughter after 1972 had approximately one less child (-0.822). The sum of the uninteracted post-1972 term and its interaction with the first child being a son is also negative, but it is smaller in magnitude than the uninteracted term (-0.4).

The results mean that parents who had their first son after 1972 were also likely

[19]This does not necessarily mean that these women had no male spouse – it could just be that the survey respondent was the oldest female in the household. To be cautious and to avoid the potentially confounding effects from having a female household head, we will control for this in our regressions.

to have had fewer children than those who had their first son before 1972 but the reduction in the number of children was smaller in magnitude than for parents that had a first daughter. This is driven by the fact that when they had a choice, i.e before 1972, many parents stopped having children once they have a son with the result that males have on average fewer siblings than females. This can be seen from the negative coefficient for the uninteracted dummy variable for whether the first child is a son.[20] All of the coefficients discussed here are statistically significant at the 1% level. In columns (5)-(8), we add controls which we will discuss in the next section.

The results in Table 2 confirm that the introduction of family planning reduced total fertility and that there is a prejudice in favor of sons. Both of these findings are important to keep in mind for interpreting our results later in the paper.

The Effect of Family Planning on Savings Next, we examine the effect of the introduction of family planning on savings. We estimate the same regressions as before, except that we replace the dependent variable with household savings rates. Table 3 shows the reduced form results. Column (1) presents the estimates when we only control for city fixed effects. On average, parents that had their first child after 1972 saved 6,175 RMB more in 2008. The estimate is statistically significant at the 1% level. In column (2), we control for basic demographic characteristics of the parents: the age of the household head and its squared term, the educational attainment of the household head and it squared term. These are important since income and consumption patterns, and thus savings patterns, can differ by age (even in our limited age range). Similarly, educated parents may have a different propensity to save relative to less educated one. Column (2) shows that including these controls have little effect on the estimated effect of having one's first child after 1972.

[20] Consistent with the stopping rule, on average, boys in our sample come from households with 1.7 children, while girls come from households with two children.

As we discussed earlier, controlling for the age of the household head introduces a specific type of selection: it raises the question of whether parents that chose to have children at an earlier time in life will save less than parents that chose to have children later in life for reasons other than the difference in total fertility. To address this, we drop the two controls for the age of the household head in column (3). The estimate is only slightly smaller than the one in column (2) and is statistically different from zero at the 1% level. The estimates in columns (2) and (3) are not statistically different from each other.

In column (4), we introduce controls for the sex of the first child and its interaction with whether he/she is born after 1972. We return to a specification where we only control for city fixed effects. The estimate of the uninteracted effect of having a first child after 1972 shows that parents that have a daughter as a first child after 1972 save 13,453 RMB more than parents that have a first daughter prior to 1972. The interaction effect shows the differential effect for parents who have their first child after 1972 but who have a son. The sum of the uninteracted and interacted effects are shown at the bottom of the table. This coefficient, 349, is positive, but small in magnitude and statistically insignificant. Thus, it means that parents that have their first child after 1972 and whose first child is a son save about the same as parents who have their first child before 1972 and whose first child is a daughter.

Given the earlier results that parents who had their first child after 1972 also had fewer children on average, these results are consistent with parents saving more when they have fewer children and in particular when the only child is a daughter.[21]

In column (5), we add the four controls for parental characteristics. In column

[21]Note that the uninteracted dummy variable for whether a first child is a son is large, positive and statistically significant. This variable, which reflects the effect of having a first child who is male prior to 1972 partly reflects the fact that such households had fewer total children because of the stopping rule (recall Table 2 column (4) shows that the coefficient of the first child being on the total number of children is -0.455), and children cost money.

(6), we remove the controls for the age of the household head and its squared term for the reasons that we discussed earlier. As before, the estimates change little with changing controls.

In column (7), we add additional controls. The control for whether the head of the household age is under 55 years of age addresses the possibility that being over the "mandatory" retirement age (from public enterprises) increases unemployment probabilities and savings behavior. Controlling for the age of the youngest child addresses the possibility that having a young child will increase consumption and affect savings. The dummy variable for whether the youngest child is under 22 years of age also addresses this point. Finally, we control for whether the mother is the household head in case this variable reflects intrahousehold bargaining power and thereby, savings behavior. In column (8), we include all of the controls in column (7) except for the age of the household head and its squared term. The estimates are precisely estimated and statistically similar to the baseline in column (5).

The estimates in Table 3 show that parents that had their first child after 1972, in particular, those with daughters, save more.

It is interesting to note that the estimates change very little with the changing controls. This is consistent with our identification assumption that the introduction of fertility restrictions was "randomly" assigned.[22]

The Implied Effect of Fertility on Savings The results in Tables 2 and 3 show that the introduction of family planning reduced fertility and increased savings, especially for parents who had a daughter as the first-born child. Together, they imply

[22]We also conduct a placebo experiment to examine the possibility that our post-1972 variable is picking up parents who prefer to have children later in life. We estimate an equation similar to equation (1), except that we replace the post-1972 dummy variable with the household head's age at first birth (both by itself and interacted with a dummy for whether the first child is a son). If our main results were driven by selection, should find the coefficient for the interaction effect to be positive. We find no effect: the coefficient is 0.00187 and the standard error is 0.00777 (these results are not reported in tables).

that lower fertility increases savings, particular for parents with only one daughter. To assess the magnitude of the effect of fertility on savings, we can instrument for the number of children and its interaction with the gender of the first child with a dummy for whether the first child was born after 1972 and its interaction with the gender of the first child. We use the 2SLS to scale the reduced form estimates from Table 3 and interpret the instrumented estimates as a rough approximation of the effect of fertility.

Since the effect of family planning on fertility is to reduce the number of children by nearly one, the magnitudes of the effects of family planning on savings rates are relatively easy to interpret (i.e., divide by negative one to approximate the instrumented effect of fertility on savings rate). In Table 4 columns (1)-(3), we report the instrumental variables estimates. The absolute value of the instrumented estimates are roughly similar in magnitude to the reduced form estimate. Column (3) shows that an additional child reduces savings by approximately 18,570 RMB if the first child is a daughter. This is statistically significant at the 1% level. The interaction effect of the number of children with a dummy for first child being male is positive and significant at the 1% level. As before, this suggests that family size matters less if the first child is male. This is shown more formally by the sum of the uninteracted and interacted effects of the number of children, which is -7,518 RMB for the level of savings in column (3). The joint estimates are statistically insignificant (they and their standard errors are not reported in the tables). We also see that the effect of the first child being male is strongly negative and significant, consistent with the theory that parents who have an oldest son expect that they will be taken care of.

Finally, we consider the alternative mechanism raised by Wei and Zhang (2011) that parents in regions with strong male-biased sex ratios and who have sons must save so that their sons can obtain brides in the future. We directly control for the interaction term of regional sex ratio and a dummy variable for whether the first child is a son (the uninteracted effect of regional sex ratio is already controlled for by the city fixed effects).[23] Our prior is that this mechanism is less relevant for our study that we study because there is little sex imbalance for these cohorts. Indeed, column (4) shows that our key results are very robust to the inclusion of this control.[24]

The Effect of Fertility on Earnings Table 5 Panel B reports on the instrumented effect of fertility on earnings and a dummy variable for whether the household head is still working (Panel A shows the reduced form estimates). This is to examine the idea that households that do not have an oldest child who is male may continue to work longer and harder to secure their old age. For brevity, we report the 2SLS estimates. Column (1) shows that an additional child results in 11,236 RMB less income in 2008 for parents if the first child is a daughter. Fertility has no effect on income for parents whose first child is a son ($-11,236 + 8,636 = -2600$), presumably because they feel secure about old-age care. Columns (2)-(7) shows that this is mainly driven by wage income.

We acknowledge that in inferring the stock of savings from the savings in one year, we must assume that the two variables are positively correlated. For example, our interpretation would be misleading if parents with fewer children accumulated more assets than parents and therefore had stopped saving. In urban China, the two main savings vehicles are savings deposits and housing. Since savings deposits generate interest income and real estate generates rental income, we can investigate this alternative explanation by examining interest income and rental income which should scale with their stock of assets. Column (5) of Table 5 shows that there is

[23] Regional sex ratio is measured as the fraction of males of those born during 1949-1975 in each city. We experimented with several alternative measures and always obtain similar results. Estimates using these other measures are available upon request.

[24] Note that the uninteracted effect of whether the first is a son is no longer meaningful by itself since it captures the effect of having a son as the first child in regions where there are no males.

no relationship between the instrumented fertility variable and interest and rental income.[25]

The Effect of Fertility on the Savings Rate While we recognize that fertility affects many aspects of people's lives (e.g., it affects both level of savings and income), for the purpose of the calibration it will be convenient to summarize the effect on fertility by a single variable, the saving rate. Since we wish to compare these results with a model where what changes is the number of children, we focus on the instrumental variables estimate. These are reported in Table 4. Columns (5) and (6) show that each additional child reduces the savings rate by eleven percentage-points. Column (7) shows that for parents with first daughters, additional children reduces the savings rate by sixteen percentage-points, while for those with sons, an additional child reduces savings rates by four percentage-points $(-0.158 + 0.118 \simeq -0.04)$. We note that the estimates on the saving rate are less precise than the estimates on savings levels. This is likely due to the fact that fertility and the sex of the eldest child also affects income. This is another reason to interpret the instrumented estimates on the saving rate as illustrative.

3.5 Interpretation

The main empirical findings are that the reduction in fertility caused by the introduction of family planning policies increased household savings, especially for parents with only one daughter. This is consistent with parents anticipating less old-age support when they have fewer children, which causes them to save more.

For the interpretation of our results and the motivation of our model in the next section, it is also important to keep in mind that parents prefer to have sons (see

[25]In our data, we also observe households own durables such as refrigerators, motorcycles, and cars; and the imputed value of housing. We find suggestive evidence that parents with children (instrumented) have, if anything, more assets than parents with fewer children. The estimates are imprecise and are available upon request.

Table 2). Consider the alternative explanation that daughters and sons provide the same level of transfers to parents, but parents with only one daughter save more because daughters cost less to raise than sons. However, this is inconsistent with the stopping rule that we see in the data (see Table 2) which suggests that parents prefer to have sons. If sons and daughters provide the same level of support and daughters cost less then parents should instead prefer to have daughters. Moreover, we note that for the cohort of urban children that we are studying, major expenditures related to child rearing (child care, housing, schooling, and even food) were state-provided. Thus, there was little cost difference between male and female children.[26] Finally, as emphasized by Wei and Zhang (2011), the tendency in China in recent years has been towards a bride price rather than a dowry, which would raise the cost of male children, though in this cohort, which predates sex-selective abortions, this effect is probably not very important either way.

Together, these findings support our interpretation that our results are driven by anticipated transfers rather than expenditures. They are consistent with qualitative and the survey evidence from Section 2 that parents see children, and particularly sons, as an importance source of old-age support.

4 A Model of Fertility and Savings

In the empirical part of this paper, we showed that the number and gender of children are important determinants of household savings. Specifically, we observed that households with more children save less. This evidence is obtained by comparing individuals who are similar except for the number of children they had: we identified the effect on the savings rate of an additional child for a household that lives in an otherwise identical economic environment. From a policy perspective, however,

[26]For example, in the 1989 UHIES, total expenditure for urban households with at least one male child was on average 1122 RMB and for households with at least one female child was on average 1129 RMB. The gap is similarly small for other years (1990-2005).

the assumption of an otherwise unchanged economic environment is unlikely to be right; a change in aggregate fertility has an impact on the economic environment, for example, through its effect on factor prices, which in turn affects savings. The micro empirical evidence cannot therefore be directly used to predict the relationship between aggregate fertility and savings. In order to address this concern, we now develop a simple overlapping generation model of savings that helps us to interpret the empirical results. We begin with the simplest version of the model to build intuition and then proceed to a more quantitative version.

4.1 The simplest OLG Model

The empirical findings that parents receive large amounts of transfers from children and that the policy-driven reduction in fertility increases household savings are consistent with the qualitative evidence that parents anticipate more transfers in expectation when they have more children. We therefore start from a variant of the classic Diamond OLG model with two additional features: (i) children transfer a fraction τ of their income to parents, (ii) parents pay a linear cost, a θ fraction of their income, to raise children. We do not model the decision to have children, but assume that every household is endowed with an exogenous number of children n_i. This choice is due to the fact that we want to consider the effect of an exogenous change in fertility, as generated by the "One-Child Policy" (or its relaxation), on savings (endogenous fertility is discussed in subsection 4.5.1). We assume log utility, a Cobb-Douglas production function and full depreciation of capital within one generation (given that a generation is twenty-five years, this is not a restrictive assumption) and that productivity grows at an exogenous rate $1 + g$. The assumption of log utility imposes that income and substitution effect perfectly offset each other, so that change in interest rate does not have any direct effect on savings. We will relax this assumption later. The economy is inhabited by a continuum of households with mass 1. Households are identical except for the number of children. Household i, with children n_i, solves the following problem

$$\max_{c_t^Y, c_{t+1}^O} log\left(c_{i,t}^Y\right) + \beta log\left(c_{i,t+1}^O\right)$$

$$s.t.$$

$$c_{i,t}^Y + \frac{c_{i,t+1}^O}{1+r_{t+1}} \leq A_t w_t \left(1-\tau-\theta n_i\right) + \frac{A_{t+1}w_{t+1}}{1+r_{t+1}}\tau n_i. \tag{3}$$

From the first order condition of this problem, we can find the household optimal saving rate, defined as $s_{i,t} \equiv \frac{A_t w_t(1-\tau-\theta n_i)-c_{i,t}^Y}{A_t w_t}$:

$$s_{i,t} = \left[\frac{\beta}{1+\beta}\right]\left[\left(1-\tau-\theta n_i\right) - \frac{\tau n_i}{\beta\left(1+r_{t+1}\right)}\left(\frac{A_{t+1}w_{t+1}}{A_t w_t}\right)\right]. \tag{4}$$

From this formula, it is clear that the model predicts that households with more children will save less. More specifically, the number of children, n_i, impacts the saving rate through two channels. First, if n_i increases, then parents have to spend more on children, so that their disposable income is reduced and consequently, they save less. We name this the "expenditure channel". An additional child decreases the saving rate by $\left(\frac{\beta}{1+\beta}\right)\theta$ through the expenditure channel. Second, if n_i increases, then parents expect to receive more transfers in old age, their need to save for retirement is therefore not as acute, which causes them to save less. We call the latter mechanism the "transfer channel". An additional child decreases the saving rate by $\frac{\tau}{(1+\beta)(1+r_{t+1})}\left(\frac{A_{t+1}w_{t+1}}{A_t w_t}\right)$ through the transfer channel.

This partial equilibrium model is able to account for the cross-households relationship between fertility and savings. However, a change in aggregate fertility has an impact on prices as well. In order to discuss how aggregate savings are affected, we therefore need to understand the aggregation and general equilibrium properties of the model.

4.1.1 General Equilibrium

In order to find the general equilibrium solution, we need to show how the model aggregates. Defining n and s to be aggregate fertility and saving rate, the following relationships hold: $n = \int n_i di$ and $s = \int s_i di$. Aggregation is trivial due to the fact that households differ only with respect to the number of children, and saving rates are linear in n_i.

The empirical results provide us with estimates of $\frac{\partial s_i}{\partial n_i}$, while, as already pointed out, we would like to have estimates of $\frac{\partial s}{\partial n}$ in order to understand the effect of the one-child policy on Chinese saving rates. To this end, we need to understand the aggregation and general equilibrium properties of the model.

We first focus on steady states. The standard law of motion of capital for the Diamond model applies to our setting and reads as

$$k_{t+1} = (1-\alpha)\frac{s_t k_t^{\alpha}}{(1+g)\, n},$$

from which we get the steady state interest rate

$$1 + r = \frac{\alpha\,(1+g)\, n}{(1-\alpha)\, s}.$$

We substitute the equilibrium interest rate into 4 and notice that, in steady state, $w_{t+1} = w_t$. Thus, we find that

$$s_i = \left(\frac{\beta}{1+\beta}\right)\left[(1-\tau-\theta n_i) - \tau\frac{n_i s}{n}\left(\frac{1-\alpha}{\alpha\beta}\right)\right]. \qquad (5)$$

Summing 5 over all households and using the fact that $s = \int s_i di$ and $n = \int n_i di$, we obtain an explicit expression for the equilibrium aggregate saving rate

$$s = \frac{\alpha\beta\,(1-\tau-\theta n)}{\alpha\,(1+\beta)+(1-\alpha)\,\tau}. \tag{6}$$

Equations 5 and 6 allow us to clearly see the difference between the partial equilibrium (PE henceforth) and general equilibrium (GE henceforth) effects of a change in fertility on savings.

The PE effect is simply the derivative $\frac{\partial s_i}{\partial n_i}$ for fixed n and s. This is given by

$$\partial_{PE} \equiv \frac{\partial s_i}{\partial n_i} = -\left(\frac{\beta}{1+\beta}\right)\left(\theta + \frac{\tau}{n}\frac{s\,(1-\alpha)}{\alpha\beta}\right).$$

We can then substitute 6 to find ∂_{PE} evaluated at equilibrium, which we name $\partial_{PE,EQ}$ and reads as

$$\partial_{PE,EQ} = -\left(\frac{\beta}{1+\beta}\right)\theta - \left(\frac{\beta}{1+\beta}\right)\left(\frac{\tau}{n}\right)\left(\frac{(1-\tau-\theta n)\,(1-\alpha)}{\alpha\,(1+\beta)+(1-\alpha)\,\tau}\right). \tag{7}$$

The GE effect is instead the derivative $\frac{\partial s}{\partial n}$, which must be computed from the equilibrium saving rate 6. This gives us

$$\partial_{GE,EQ} \equiv -\left(\frac{\beta}{1+\beta}\right)\theta\frac{\alpha\,(1+\beta)}{\alpha\,(1+\beta)+(1-\alpha)\,\tau}. \tag{8}$$

Comparison of PE and GE effects

We now compare the difference between the PE and GE effects of an increase of fertility on saving rates. First let's notice that $\partial_{PE,EQ}$ is made of two parts: (i) $\partial_{PE,Expend} \equiv -\left(\frac{\beta}{1+\beta}\right)\theta$ and (ii) $\partial_{PE,Transf} \equiv -\left(\frac{\beta}{1+\beta}\right)\left(\frac{\tau}{n}\right)\left(\frac{(1-\tau-\theta n)(1-\alpha)}{\alpha(1+\beta)+(1-\alpha)\tau}\right)$. Part (i) is the expenditure channel: an additional child decreases savings due to the fact that current income is reduced by direct expenses for child support. Part (ii) is the transfer channel: an additional child increases the transfers received while retired so that households can afford to save less.[27] The transfer channel, $\partial_{PE,Transf}$, is equal to zero when $\tau = 0$, while it is negative for all other admissible values of τ.

Second, notice that $\partial_{GE,EQ}$ can be rewritten as

$$\partial_{GE,EQ} = \partial_{PE,Expend}\varphi(\alpha, \beta, \tau),$$

where $\varphi(\alpha, \beta, \tau) \leq 1$ for all parameters and is equal to 1 only if $\tau = 0$. From this last equation we see that absent any transfer from children to parents (i.e., $\tau = 0$), $\partial_{GE,EQ} = \partial_{PE,EQ}$ because the expenditure channel is identical in PE and GE. In contrast, for any positive τ, $\partial_{GE,EQ} > \partial_{PE,EQ}$, so that the effect of an additional child on saving is smaller in GE than in PE.

Discussion

PE and GE effects are different for two reasons: (i) in GE, the transfer channel is muted, so that $\partial_{GE,Transf} = 0$; and (ii) in GE, the expenditure channel is smaller than in PE, which is given by $\varphi(\alpha, \beta, \tau) \leq 1$.

Let's first discuss (i). An additional child provides a benefit in the future: parents need to save less today because they are expecting to receive more transfers from children when retired. The present value of these future transfers is lower if the interest rate is higher. This is what Summers (1981) called a wealth effect, to distinguish it from the income effect of increasing the interest rate, which exactly offsets the substitution effect in this log utility case. In GE, an increase in aggregate fertility raises the interest rate and under the assumptions of log utility and full

[27]Note that an additional child provides a negative income shock through channel (i), while it provides a positive income shock through channel (ii). Our interpretation of the timing of this model is that the negative income shock happens when the household is saving while the positive income shock happens when the household is dissaving. It is true that when we observe these families their children are grown-ups and typically are beyond the age when they need investments. The interpretation of the expenditure effect therefore rests on the idea that households spent more on their children when their children were young, thus postponing other expenditures (house purchase, house repair, etc.) till they were older.

depreciation, this consequent reduction in the value of the transfer exactly offsets the direct impact of increased fertility on total transfers. As a consequence, the transfer channel is effectively turned off: $\partial_{GE,Transf} = 0$.

Next, we discuss (ii). The expenditure channel does not directly depend on the interest rate. This is because both the spending on children and the savings decision are made in the same period. However, in GE, the direct effect of an additional child on spending reduces aggregate savings, which then implies capital scarcity and higher interest rates. The resulting reduction in the value of future transfers leads, as before, to higher savings, which partly compensates for the reduction in savings coming from the expenditure channel. This is why we find that $\varphi(\alpha, \beta, \tau) \leq 1$. Obviously, when there are no transfers from children ($\tau = 0$), this effect is shut down and $\varphi(\alpha, \beta, \tau) = 1$.

Out of Steady-State Dynamics

So far, our focus has been on steady states. We now show that the previous results, and in particular the important role that general equilibrium forces have on the relationship between fertility, transfers and savings, hold on the transition path from one steady state to another. The only change that occurs when we go onto the transition path is that there is a wage effect as well as an interest rate effect, with wage growth slowing down (relative to steady state trend) and the interest rate going up as the labor force grows (because of increased fertility). Both of these effects encourage parents to save more: the interest rate effect for reasons already discussed and the wage effect because lower children's earning means lower transfers in the future.

More formally, we can substitute the equilibrium expression for interest rate, $1 + r_{t+1} = \alpha k_{t+1}^{1-\alpha}$, and wage, $w_t = (1-\alpha) k_t^{\alpha}$, in the formula for the saving rate to obtain:

$$s_{i,t} = \left(\frac{\beta}{1+\beta}\right)\left[(1-\tau-\theta n_{i,t}) - \frac{\tau n_i}{\alpha\beta}(1+g)\frac{k_{t+1}}{k_t^{\alpha}}\right].$$

We can further manipulate this expression, substituting the law of motion of capital, which must hold even out of steady state, and summing over all households in order to solve for the aggregate saving rate on the transition path

$$s_t = \frac{\alpha\beta(1-\tau-\theta n_{t+1})}{\alpha(1+\beta)+\tau(1-\alpha)}.$$

This formula exactly mirrors the steady state formula 6, such that $\frac{\partial s_t}{\partial n_{t+1}} = \frac{\partial s}{\partial n}\ \forall t$. In other words, in this example with full depreciation and log preferences, being on the path to a steady state is identical with being at the steady with respect to how fertility affects savings. This is because the smaller rise in interest along the transition path (because capital does not jump to its new steady state value) is compensated by the reduction in wage growth (which dissipates when we reach the new steady state).

4.2 Generalizing the model

In order to bring the model closer to the data, we now add a richer set of demographic features and relax the assumption of log utility in favor of a CRRA utility function.

Demographics We introduce two new elements into the previous model: (i) we allow a household to include a father and a mother, both of whom transfer to their own parents; (ii) we distinguish between sons and daughters, to match the fact that parents rely more on sons than daughters for old age support. We assume that males and females earn the same.[28] However, daughters transfer a fraction $\lambda < 1$ of what

[28] We could in principle allow for earnings to be different between men and women by adjusting the relative shares of income transferred by men and women.

sons transfer to their parents.[29] Following the empirical evidence discussed earlier, we assume that the cost of raising children is the same whether they are a boy or a girl.

Accounting for these demographic characteristics, the budget constraint 3 becomes

$$c_{i,t}^{Y}+\frac{c_{i,t+1}^{O}}{1+r_{t+1}}\leq 2A_{t}w_{t}\left(1-\tau\left(1+\lambda\right)-\theta\left(n_{i}^{m}+n_{i}^{f}\right)\right)+\frac{A_{t+1}w_{t+1}}{1+r_{t+1}}\tau\left(n_{i}^{m}+\lambda n_{i}^{f}\right),$$

where n_i^m is the number of sons in household i and n_i^f is the number of daughters in household i.

CRRA Utility Function To allow households to have an inter-temporal elasticity of substitution different than one, we use a CRRA utility function, $u(x)=\frac{x^{1-\rho}}{1-\rho}$, where $\frac{1}{\rho}$ is the inter temporal elasticity of substitution (IES). If $\rho>1$, then the IES is smaller than 1, which implies that an increase in the interest rate decreases savings because the substitution effect is weaker than the income effect. $\rho=1$ gives the log utility case already analyzed.

4.2.1 Some Intuition for this Case

We solve the first order conditions of the model with the new budget constraint and the CRRA utility to obtain the saving rate for household i

$$s_{i,t}=\left[\frac{\beta^{\frac{1}{\rho}}\left(1+r_{t+1}\right)^{\frac{1-\rho}{\rho}}}{1+\beta^{\frac{1}{\rho}}\left(1+r_{t+1}\right)^{\frac{1-\rho}{\rho}}}\right] \tag{9}$$

$$\left[\left(1-\tau\left(1+\lambda\right)-\theta\left(n_{i}^{m}+n_{i}^{f}\right)\right)-\frac{\tau\left(n_{i}^{m}+\lambda n_{i}^{f}\right)}{\beta^{\frac{1}{\rho}}\left(1+r_{t+1}\right)^{\frac{1}{\rho}}}\left(\frac{A_{t+1}w_{t+1}}{2A_{t}w_{t}}\right)\right]. \tag{10}$$

To build some intuition, we sum 9 over all households and using the formula

[29] We could alternatively assume that females earn a fraction λ of males and transfer the same proportion of their income to parents.

for the steady state interest rate, which is unchanged by the new assumptions. We obtain a formula for the steady state aggregate saving rate

$$s = \left(\frac{\beta^{\frac{1}{\rho}} \left(\frac{\alpha(1+g)\left(n^m+n^f\right)}{(1-\alpha)s} \right)^{\frac{1-\rho}{\rho}}}{1 + \beta^{\frac{1}{\rho}} \left(\frac{\alpha(1+g)\left(n^m+n^f\right)}{(1-\alpha)s} \right)^{\frac{1-\rho}{\rho}}} \right) \left(\left(1 - \tau(1+\lambda) - \theta\left(n^m + n^f\right)\right) - \frac{\tau\left(n^m + \lambda n^f\right)}{\beta^{\frac{1}{\rho}} \left(\frac{\alpha(1+g)\left(n^m+n^f\right)}{(1-\alpha)s} \right)^{\frac{1}{\rho}}} \frac{(1+g)}{2} \right),$$

where n^m and n^f are the aggregate numbers of sons and daughters fertility, and s is the aggregate saving rate. The steady state saving rate is the product of two square bracketed terms. Within the second bracket, the first term is the cost of an extra child and the second term captures the fact that an extra child brings more future income and hence reduces savings. In GE, these two partial equilibrium effects are augmented by two more effects, both operating through the denominator of the second term. The first is the wealth effect resulting from the increase in the interest rate caused by the increase in fertility. The second is the feed-back from the increase in savings, which pushes the interest rate down and therefore mitigates the wealth effect.

Then there is the first square bracket, which captures the income and substitution effects resulting from the increase in the interest rate. Assuming that $\rho > 1$ (we later argue that this is the interesting case), the increase in the interest rate induced by the increase in fertility must reduce the part of savings that is determined by the income and substitution effects. This reduction in savings in turn has a feedback effect which further raises the interest rate and further reduces savings. This positive feedback loop is the reason why the GE effect can be larger than the PE effect. We will provide some examples when we present the quantitative model.

4.3 Using the Micro Evidence to Identify Model Parameters

In this section, we use the micro empirical evidence from earlier to pin down some of the key parameters of the model so that we can predict the GE relationship between fertility and savings. The regressions from Section 3.4 give us two coefficients that are useful for identifying the relative magnitude of the expenditure and transfer channels. The results in Table 4 column (7) show two relationships: *i*) that households with only one son save on average approximately 10 percentage-points less than households with only one daughter; and *ii*) that households with two children save on average approximately 10 percentage-points less than households with only one child.[30] These coefficients are admittedly not all very precisely estimated. Thus, in section C in the Online Appendix, we conduct a robustness exercise to demonstrate that our results are not sensitive to reasonably different parameter values. Finally, note that the average saving rate in our sample, which allows us to pick the discount factor β, is 26 percentage-points.

Empirical results (i) and (ii) identify the contributions of the expenditure and transfer channels to savings as a function of the parameter λ, which captures the relative transfers of a daughter as a function of those of a son. As an intermediate step, it is useful to redefine the expenditure and transfer channels in the complete model[31]. We call the two channels $\tilde{\partial}_{PE,Expend}$ and $\tilde{\partial}_{PE,Transf}$ to distinguish them

[30]The coefficients are the following: # kids -0.158, # kids x 1st is male 0.116, 1st is male -0.215. Ignoring the constant, fixed effects and controls in the regression, the predicted savings rates for households with different numbers and sexes of children are the following: 1 son $-0.158 + 0.118 - 0.215 = -0.255$, 1 daughter -0.158, 1 son + 1 other child $2(-0.158) + 2(0.118) - 0.215 = -0.295$, 1 daughter + 1 other child $2(-0.158) = -0.316$. Thus, the difference in savings rate between a household with only one son and only one daughter is $-0.255 - (-0.158) \approx 0.1$, and the difference between households with two children and households with one child is around 0.099, which is the average of $-0.295 - (-0.255), -0.295 - (-0.158), -0.316 - (-0.255), -0.316 - (-0.158)$.

[31]The introduction of CRRA utility slightly alters the formula for the expenditure and transfer channels, which now both depend on the values of the IES (Inter temporal Elasticity of Substitution) and the interest rate. Note that in partial equilibrium, we can decompose the effect of an additional child on savings to the direct effect from higher immediate expenditures (the expenditure channel)

from the formula for the simplest model. They are given by

$$\tilde{\partial}_{PE,Expend} \equiv \left[\frac{\tilde{\beta}_{t+1}\left(1+r_{t+1}\right)^{-1}}{1+\tilde{\beta}_{t+1}\left(1+r_{t+1}\right)^{-1}}\right]\theta \tag{11}$$

$$\tilde{\partial}_{PE,Transf} \equiv \left[\frac{\tilde{\beta}_{t+1}}{1+\tilde{\beta}_{t+1}\left(1+r_{t+1}\right)^{-1}}\right]\left(\frac{A_{t+1}w_{t+1}}{2A_t w_t}\right)\tau. \tag{12}$$

where we have defined $\tilde{\beta} \equiv \beta^{\frac{1}{\rho}}\left(1+r_{t+1}\right)^{\frac{1}{\rho}}$In order to identify $\tilde{\partial}_{PE,Transf}$, we use empirical result (i). According to the model, the difference in the saving rate between a household with only one daughter and a household with only one son is given by $(1-\lambda)\,\tilde{\partial}_{PE,Transf}$. Hence, using the empirical evidence, we have that

$$0.10 = (1-\lambda)\,\tilde{\partial}_{PE,Transf}, \tag{13}$$

which identifies $\tilde{\partial}_{PE,Transf}$ as a function of λ.

In order to identify $\tilde{\partial}_{PE,Expend}$, we use empirical result (ii). According to the model, the difference in the saving rate between a household with one child and a household with two children is given by $\tilde{\partial}_{PE,Expend} + \frac{1}{2}(1+\lambda)\,\tilde{\partial}_{PE,Transf}$. Hence, using the empirical evidence, we have that

$$0.10 = \tilde{\partial}_{PE,Expend} + \frac{1}{2}(1+\lambda)\,\tilde{\partial}_{PE,Transf}. \tag{14}$$

Equations 13 and 14 can be solved to obtain values for the expenditure and transfer channels as a function of λ:

$$\tilde{\partial}_{PE,Expend} = 0.10\left(1-\frac{1}{2}\left(\frac{1+\lambda}{1-\lambda}\right)\right)$$

and the indirect effect from expected future transfers (the transfer channel). Thus, we can still consider the formula under CRRA as capturing the expenditure and transfer effects. Taking GE effects into account will affect savings through both the channels.

$$\tilde{\partial}_{PE,Transf} = 0.10\left(\frac{1}{1-\lambda}\right).$$

It is immediately obvious that $\tilde{\partial}_{PE,Expend}$ is decreasing in λ, while $\tilde{\partial}_{PE,Transf}$ is increasing in λ. Intuitively, if λ is close to one, parents expect similar transfers from daughters and sons. For parents with sons and daughter to have very different savings, the level of the transfers must be high enough to magnify the relatively small gender difference in transfer rates into large differences in transfers and hence savings, which implies that $\tilde{\partial}_{PE,Transf}$ itself must be very large. If $\tilde{\partial}_{PE,Transf}$ is large, all of the difference in savings between households with one and two children will be driven by the expectation of future transfers, and the expenditure channel will be of limited relevance, which explains why $\tilde{\partial}_{PE,Expend}$ is decreasing in λ.

Since it is costly to raise children, we assume that $\tilde{\partial}_{PE,Expend} \geq 0$. This restriction implies that $\lambda \in \left[0, \frac{1}{3}\right]$, which is consistent with the stylized evidence from Section 2 that daughters transfer considerably less than sons. The range of $\lambda \in \left[0, \frac{1}{3}\right]$ corresponds to $\tilde{\partial}_{PE,Expend} \in \left[0, \frac{1}{2}\tilde{\partial}_{PE,Transf}\right]$ – i.e., the empirical evidence implies that the transfer channel will dominate the expenditure channel.

Next, we want to solve for the primitive parameters θ and τ. To do this, we need to pin down a few additional parameters. In particular, equations 14 and 13 show that we need to choose values for $\frac{A_{t+1}w_{t+1}}{A_t w_t}$,$1+r$, and $\tilde{\beta}$. We calculate $\frac{A_{t+1}w_{t+1}}{A_t w_t}$, the growth rate of wage income, from the UHS data. We use the average real deposit rate in China as the value for r. **?** reported that the average real deposit rate in China between 1998 and 2012 is equal to 0.91%. We use their estimate. We then notice that the average saving rate is strictly increasing in $\tilde{\beta}$ and we thus pick $\tilde{\beta}$ in order to match the average saving rate in our data, which is equal to 26%. In order to calculate the average saving rate, we need to pick a value for the average number of children. We use $n = 1.88$, which is the average number of children in the sample used for our regression analysis. Then, for a given value of λ, we can calculate the corresponding values of θ, τ.

In Table 6, we report the estimated parameter values for the two extreme case of $\lambda = 0$ and $\lambda = \frac{1}{3}$. The value of τ implies that an adult male transfer between 8% and 15% of his income to his parents. This is consistent with the UHIES data, which report that total transfer expenditures is approximately 8% of total household income for the average household with a male household head between 25 to 40 years of age. Our estimated value of τ is thus consistent with the limited empirical evidence[32].

The value of θ is estimated to be no more than 10%, which implies that every child costs no more than 10% of household income. This is roughly consistent with the data reported by the *China Health and Nutritional Survey*, which shows that an urban household in 1989 spends approximately 8% of total income on food, clothing and schooling for children.[33]

The value of β depends on the the value of ρ, and thus varies around calibration. For our preferred estimates, the ones with $\rho = 1$, β is equal to 0.995: in order to match the high saving rate we need individuals to be quite patient.

4.4 Quantitative Results

Equipped with the estimates of the primitive parameters, we can now quantify how the GE effect relates to the PE effect. But before doing so, we need to discuss how we deal with the interest rate within the model. In the calibration exercise, we have used the market interest rate that households face on deposits. The model has instead a prediction for the marginal product of capital. The marginal product of

[32]We acknowledge that assessing the plausibility of the transfer rate is difficult. To the best of our knowledge, there is no reliable data on transfers to parents at the individual level. Moreover, the ability of children to insure old parents in bad states of the world and cohabitation during old age is likely to be very valuable to parents and is difficult to measure or monetarize.

[33]This result must be interpreted with caution. The fact that our empirical results use a sample of individuals age 50 to 65 who spend less on children than younger parents who have younger children means that our results could underestimate the effect of the expenditure channel. In light of this, the benchmark exercise considers the case where parental expenditures on children, θ, takes the maximum value (i.e., $\lambda = 0$). Appendix Section C further explores the sensitivity of our calibration results to alternative parameter values.

capital implied by the model in the baseline equilibrium, the one with $s = 0.26$ and $n = 1.88$, is not equal to the observed returns on savings in China. We thus need to calibrate a last parameter, that is the wedge between the marginal product of capital and the interest rate that households face on savings. We call this wedge ψ, which solves $1+r = \psi\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{n}{s}\right)(1+g)\right]$, where the left hand side of the equation is the market interest rate in China, as previously discussed, and the right hand side is the wedge multiplied by the marginal product of capital in equilibrium as a function of saving rate and fertility, evaluated at the baseline parameters of $n = 1.88$ and $s = 0.26$. We assume that ψ is invariant to policies that affect fertility and we thus keep it constant throughout the counterfactual experiments, so that changes in n and s are going to be reflected into changes of the interest rate that households face. Given this setup, we can vary the exogenous level of fertility n, and solve for the endogenous saving rate s that is predicted by the model.

Using this simple procedure, we can compute the hypothetical aggregate saving rates that the model implies for any value of n. We repeat the same procedure for different values of ρ between 0.5 and 3. In Figure 2, we plot aggregate saving rates as a function of aggregate fertility for the case in which $\lambda = 0$. In Figure 3, we repeat the same exercise for the case in which $\lambda = \frac{1}{3}$. For comparison purposes, we include the PE relationship between fertility and savings in the figure, which is from the earlier empirical estimates.

Figure 2 is the case where $\lambda = 0$, such that daughters transfer nothing. The red line displays the partial equilibrium relationship, which is the observed saving rates of households in the same economy and have a different number of children. The black solid line displays the general equilibrium saving rates that are implied by different level of aggregate fertility when $\rho = 1$. It shows the saving rate that the model predicts for a hypothetical situation in which all households would change their fertility level. The black line is flatter than the red line. This implies that an increase in aggregate fertility has a smaller effect on savings than the one that we estimated comparing different households.

The difference between partial and general equilibrium is large: a household that has one additional child on average saves ten percentage-points less; but if all households have one additional child, the aggregate saving rate decreases by only 3.3 percentage-points. The additional lines in the figure display aggregate saving rates for different values of ρ. As noted earlier, if ρ is larger than one, then the general and partial equilibrium effects are more similar. For very high values of ρ, it is even possible for the GE effect to be stronger than the PE effect. For example, as shown in Figure 2, if we consider the case with $\rho = 3$, then due to a very strong income effect, the GE effect would be larger than the PE effect. However, it is worth mentioning that $\rho = 3$ is an extreme value within the set of accepted estimates of ρ.

Figure 3 is identical to the previous one, but uses the parameters estimated assuming that $\lambda = \frac{1}{3}$. When $\lambda = \frac{1}{3}$, the expenditure channel is completely shut down (since it implies $\theta = 0$), which means that an increase in aggregate fertility has no effect on aggregate savings for when $\rho = 1$. This is why in the figure, the black GE line is flat at 26 percentage-points. In general, increasing λ magnifies the estimated difference between partial and general equilibrium effects.

Finally, we need to say something about the value of ρ. There is no consensus in the literature. A recent survey of the literature by Attanasio and Weber (2010) argues that ρ is reasonably around 1.5. If $\rho = 1.5$, then extrapolating from the PE evidence to predict the effect of an aggregate increase in fertility would overestimate the increase in saving rate by as much as 50% even in the most conservative calibration (the one with $\lambda = 0$).

In summary, the quantitative analysis shows that extrapolating from the partial equilibrium evidence to predict that effect of the removal of the one child policy is likely to significantly overestimate the effect of the increase in fertility on savings.

4.5 Further Generalizations

Thus far, we have considered a model with only the minimal structure necessary to match the empirical results and be able to conduct the GE counterfactual. We now explore the implications of extending the model along three dimensions: endogenous fertility, endogenous human capital investment and endogenous transfers. We show if and how they change the difference in the partial and general equilibrium effects of fertility on savings. The discussion follows the baseline model from Section 4.1 and focuses on the key intuitions.[34]

4.5.1 Endogenous Fertility

In the baseline model, we have assumed that parents do not decide how many children to have. This assumption fits our purpose both because the change in fertility is exogenous in the partial equilibrium empirical estimates and also because the general equilibrium counterfactual aims to find the effect of an exogenous increase in aggregate fertility, as generated by the relaxation of the one-child policy, on savings.

If we assumed instead that parents treat children as an investment goods (e.g., Caldwell, 1982; Boldrin and Jones, 1988) (as against a consumption good, as in (e.g., Becker and Barro, 1988). The decision to have a child is an investment that has an immediate cost (from raising the child), and entails the future benefit of transfers that are received from the adult child. Suppose that parents can invest in two assets–children and savings and try to optimize their portfolio across these two assets.[35]

[34]Section B in the appendix provides a more formal and detailed description.

[35]We use the word invest to indicate actions that move wealth from one period to the next. In the context of our model, households invest to have income available for when they retire. We use instead the word savings to indicate uniquely investment in monetary instruments, for example in a bank account. We adopt this distinction due to the fact that in our model households can invest in both children or savings.

Because of the "lumpiness" of the number of children, not everyone will invest necessarily in the same number of children; at the optimum, some otherwise identical families will have more savings and others will have more children. In the cross-section of families the correlation between savings and the number of children will be negative.

Now suppose a new regulation is introduced which restricts the preferred number of children to be below a certain cutoff. For the households for whom this constraint is binding, the number of children will go down and the savings will go up in partial equilibrium. This is very similar to our analysis of an exogenous change in the number of children above.

However in GE there are two more effects that we did not have previously: First, wages would be expected to go up faster than productivity for some time, and this might induce some unconstrained households to increase their fertility. This will counteract the effect of the regulation. Second, interest rates will go down, making investment in children relatively more attractive and this again would push the unconstrained households to have more children.

This analysis, which is formalized in section B in the appendix, shows that in the presence of endogenous fertility, the partial and general equilibrium relationships between fertility and savings are likely to be quite different.

4.5.2 Human Capital Investment

Our model has thus far ignored human capital investment. In this section, we discuss the implications of allowing parents to invest in the human capital of their children. In the model, parents are willing to invest in their children's human capital in anticipation of higher future transfers – i.e., investment in children's human capital increases their future wages and as a consequence anticipated transfers. Children's education is thus an investment, which requires an upfront cost but pays a benefit in the form of higher expected transfers.

We begin by assuming that there is no quantity-quality trade-off in partial equilibrium, such that parents' investment in their children's human capital is independent from the number of children itself. Even in the absence of a trade-off in partial equilibrium, a quantity-quality trade-off emerges in general equilibrium: increased *aggregate* fertility causes a reduction in human capital investment per child. The reason is that higher aggregate fertility increases the interest rate, which reduces the value of transfers and thus the incentives for parents to invest in children's education. The decrease in human capital investment implies that as fertility rises, expenditure per child decreases, and thus savings increase. Under the assumptions of the model in Section 4.1, the decrease in human capital caused by the higher interest rates is sufficiently strong to fully compensate the expenditure channel. The consequence is that in GE, fertility and savings are not related.

Now let us also assume the presence of a partial equilibrium quantity-quality tradeoffs such that households with more children invest less per child in human capital. The partial equilibrium quantity-quality tradeoff does not have corresponding effect in the steady state of the general equilibrium economy, due to the fact that an increase in fertility also reduces the human capital of parents and thus the opportunity cost of raising children.

These two results taken together imply that the introduction of endogenous human capital investment makes the difference between partial and general equilibrium results even larger, and thus cannot overrule our main qualitative result or the quantitative results discussed in Section 4.4. The arguments are formally presented in section B in the appendix.

4.5.3 Endogenous Transfers

In the baseline model, we assume that transfers to parents are exogenously determined. We could alternatively extend our model along the lines of Boldrin and Jones (2002) and assume that children make transfers because they care about their parents well-being. If we allow transfers rate to be endogenously determined, the partial and general equilibrium relationships between fertility and savings becomes even more different. When transfer rates are endogenous, increasing the number of children reduces transfer rates per child. This occurs for three different reasons. First, the increase in fertility decreases the incentive of each child to transfer to parents due to the strategic interactions among siblings. Second, it implies that young individuals must spend more on child rearing, and thus, they transfer less to their own parents. Third, an increase in aggregate fertility increases the interest rate, which reduces the value of transfers to parents, and thus reduces the incentives of altruistic children to make transfers.

The first two reasons are present both in partial and general equilibrium, while the third one emerges only due to the effect of fertility on the interest rate: in general equilibrium the negative relationship between the number of children and transfer rate is stronger. As aggregate fertility increases, the total transfers received from children increase less than proportionally because each child transfers less. Therefore, parents save more relative to the case with exogenous transfer rates. Allowing for endogenous transfer rate thus magnifies the difference between partial and general equilibrium results. The arguments are formally presented in section B in the appendix.

5 Conclusion

The goal of this paper is to illustrate the challenges of using partial equilibrium estimates of behavioral parameters to analyze the effects of policies that affect the full equilibrium of the economy, but also the rewards of using them in combination with a model to infer what the full equilibrium effect would be. In the world described by the our model, the partial equilibrium effects of demographic changes substantially overestimate the full equilibrium effect. This is important to document because a great deal rides on what we think will happen as a result of the end of phenomena such as China's One Child Policy or Japan's demographic collapse.

At the same time, our study highlights the sensitivity of the model-derived quantitative effects to the parameters that are used. Thus, an important endeavor for future studies on the effect of aggregate fertility change is to obtain reliable parameter estimates from careful micro-empirical estimates.

There are, of course, many caveats to keep in mind in interpreting our main result. Most importantly, rational expectations about the relatively distant future plays an important role in our argument. In our model, parents react to the fact that the current boom in fertility will raise interest rates in the future when these children join the labor force. In contrast, if parents do not make the connection between current fertility changes and future price changes, the partial equilibrium predictions would be the right ones. Finding reliable evidence that helps us determine the plausibility of this assumption remains a very important part of this research agenda.

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Table 1: Means

Variable	Obs	Mean	Std. Dev.
Income	475	49583.57	37070.81
Expenditure	475	32421.29	26117.09
Savings (Income-Expenditure)	475	17162.28	25361.96
Savings Rate (Savings/Income)	475	0.26	0.29
# Kids	475	1.88	0.84
Fraction male	475	0.498	0.38
Year of Birth of First Child	475	1973	2.94
Year of Birth of the Last Child	475	1976	4.23
Age of Household Head	475	60.66	3.03
Years of Education for the HH Head	475	9.73	1.49
Fraction of Female HH Heads	475	0.42	0.49

Table 2: The Effect of Family Planning on Fertility

	Dependent Variable: # Kids							
	(1)	(2)	(3)	(4)	(5) Baseline	(6)	(7)	(8)
Dep. Var Mean	**1.88**	**1.88**	**1.88**	**1.88**	**1.88**	**1.88**	**1.88**	**1.88**
1st Born 1972+	-0.589	-0.521	-0.581	-0.822	-0.754	-0.812	-0.913	-0.997
	(0.106)	(0.098)	(0.064)	(0.010)	(0.027)	(0.012)	(0.030)	(0.029)
1st Born 1972+ x 1st is a son				0.417	0.416	0.413	0.375	0.369
				(0.085)	(0.083)	(0.085)	(0.090)	(0.092)
1st is a Son				-0.455	-0.454	-0.454	-0.398	-0.398
				(0.064)	(0.061)	(0.062)	(0.066)	(0.068)
Controls								
HH Head Age	N	Y	N	N	Y	N	Y	N
HH Head Age Squared	N	Y	N	N	Y	N	Y	N
HH Head Years of Edu	N	Y	Y	N	Y	Y	Y	Y
HH Head Years of Edu Squares	N	Y	Y	N	Y	Y	Y	Y
HH Head Age >55	N	N	N	N	N	N	Y	Y
Age of Youngest Child	N	N	N	N	N	N	Y	Y
Youngest Child Age < 22	N	N	N	N	N	N	Y	Y
Mother is HH Head	N	N	N	N	N	N	Y	Y
Observations	475	475	475	475	475	475	475	475
R-squared	0.265	0.276	0.270	0.302	0.313	0.307	0.417	0.405
Joint: 1st Born 1972+ 1st Born 1972 x 1st is a Son				-0.405	-0.338	-0.398	-0.539	-0.628
p-value				0.000	0.000	0.000	0.000	0.000

Notes: All estimates control for city fixed effects. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).

Table 3: The Effect of Family Planning on Household Savings

	Dependent Variables: Savings							
	(1)	(2)	(3)	(4)	(5) Baseline	(6)	(7)	(8)
Dep. Var Mean	**17162**	**17162**	**17162**	**17162**	**17162**	**17162**	**17162**	**17162**
1st Born 1972+	6,175	7,355	5,673	13,453	14,361	12,730	13,466	11,417
	(3,689)	(4,065)	(2,366)	(470)	(1,042)	(512)	(1,226)	(968)
1st Born 1972+ x 1st is a son				-13,104	-12,613	-12,692	-12,979	-13,119
				(2,687)	(2,638)	(2,663)	(2,718)	(2,710)
1st is a Son				11,097	10,989	10,981	11,795	11,783
				(1,741)	(1,701)	(1,715)	(1,868)	(1,870)
Controls								
HH Head Age	N	Y	N	N	Y	N	Y	N
HH Head Age Squared	N	Y	N	N	Y	N	Y	N
HH Head Years of Edu	N	Y	Y	N	Y	Y	Y	Y
HH Head Years of Edu Squares	N	Y	Y	N	Y	Y	Y	Y
HH Head Age >55	N	N	N	N	N	N	Y	Y
Age of Youngest Child	N	N	N	N	N	N	Y	Y
Youngest Child Age < 22	N	N	N	N	N	N	Y	Y
Mother is HH Head	N	N	N	N	N	N	Y	Y
Observations	475	475	475	475	475	475	475	475
R-squared	0.106	0.129	0.125	0.125	0.147	0.144	0.158	0.152
Joint: 1st Born 1972+ 1st Born 1972 x 1st is a Son				348.4	1748	37.94	487.2	-1703
p-value				0.893	0.521	0.988	0.871	0.543

Notes: All estimates control for city fixed effects. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).

Table 4: The Effect of Fertility on Household Savings

	Dependent Variables							
	Savings				Savings/Income			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dep Var Means	**17162**	**17162**	**17162**	**17162**	**0.26**	**0.26**	**0.26**	**0.26**
# Kids	-14122	-14155	-18571	-18574	-0.110	-0.110	-0.158	-0.158
	(5828)	(5780)	(1752)	(1741)	(0.063)	(0.062)	(0.028)	(0.028)
# Kids x 1st is a Son			11052	11163			0.118	0.116
			(6846)	(6806)			(0.098)	(0.098)
1st is a Son		803	-19910	-30358		0.007	-0.215	-0.026
		(1723)	(12401)	(15853)		(0.025)	(0.184)	(0.261)
Controls								
Regional Sex Ratio x 1st is a Son	N	N	N	Y	N	N	N	Y
Observations	475	475	475	475	475	475	475	475
R-squared	0.008	0.008	0.001	0.003	0.018	0.018	0.006	0.007
F-stat (1st Stage)	53.51	55.86	20.02	19.94	53.51	55.86	20.02	19.94

Notes: All estimates control for baseline controls: city fixed effects, age of the household head and its squared term, education of the household head and its squared term. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The excluded instruments are: a dummy variable for whether the first child is born after 1972, and the interaction terms between whether the first child is male and dummy variables for if the first child is born after 1972. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).

Table 5: The Effect of Fertility on Income

	Dependent Variable: Income						
	(1) Total	(2) Wages	(3) Other Labor	(4) Business	(5) Interest & Rent	(6) Pension	(7) Welfare
Dep Var Means	**49584**	**19708**	**120.1**	**2132**	**1800**	**25650**	**288.8**
# Kids	-11,235.635	-8,803.966	-65.054	-1,851.312	-411.131	-211.735	60.291
	(1,973.216)	(1,637.266)	(123.057)	(726.407)	(394.466)	(718.815)	(169.558)
# Kids x 1st is a Son	8,636.034	-2,890.272	-897.833	5,317.645	1,397.633	3,020.509	1,760.605
	(9,044.960)	(8,178.309)	(450.710)	(4,210.454)	(1,850.359)	(4,090.164)	(1,083.093)
1st is a Son	-11,361.338	5,893.019	1,778.735	-7,844.857	-2,742.532	-3,919.576	-2,679.567
	(16,015.243)	(14,710.105)	(868.134)	(7,383.570)	(3,540.745)	(7,214.946)	(1,766.620)
Observations	475	475	475	475	475	475	475
R-squared	0.280	0.126	-0.081	0.160	0.198	0.346	-0.079

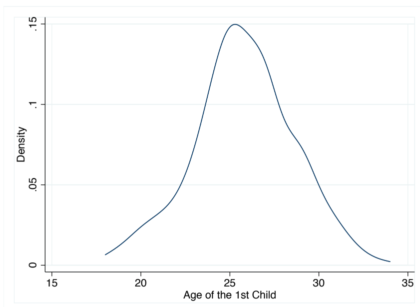
Notes: The excluded instruments are the dummy variable for the 1st child born in 1972+ and its interaction with whether the first child is male. All estimates control for baseline controls: city fixed effects, age of the household head and its squared term, education of the household head and its squared term. Standard errors, clustered at the level of birth year-sex-city are presented in parentheses. There are 131 clusters. The sample uses households that have their first child during 1967-77, and where the age of the household head is 50-65. Source: RUMiC (2008).

Table 6: Parameter Values

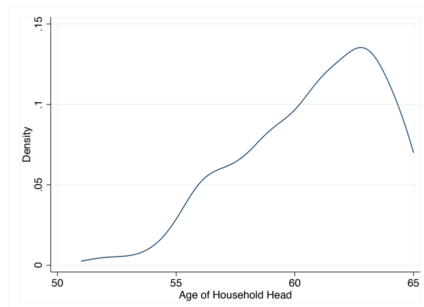
	$\lambda = 0$	$\lambda = \frac{1}{3}$
θ	10.18%	0%
τ	8.77%	15.32%

Figure 1: Distribution of Age and Savings in RUMiC Sample– Kernel Density with Gaussian Kernel Function

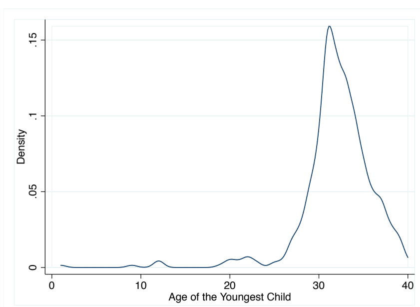
(a) Age of the First Child



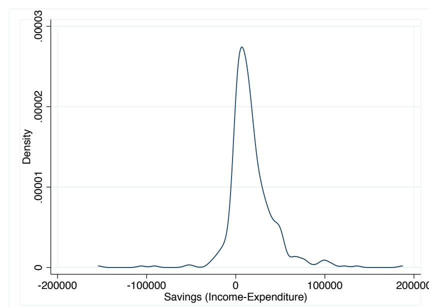
(b) Age of the Household Head



(c) Age of the Youngest Child



(d) Savings



(e) Savings Rate

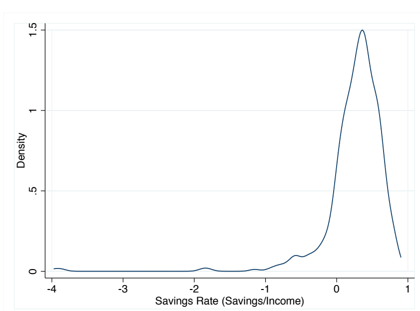


Figure 2: PE vs GE for $\lambda = 0$

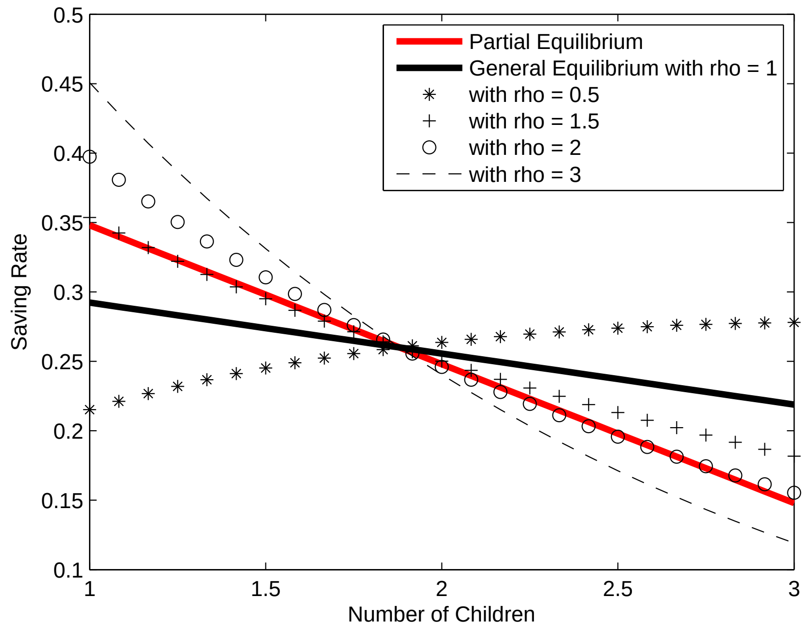
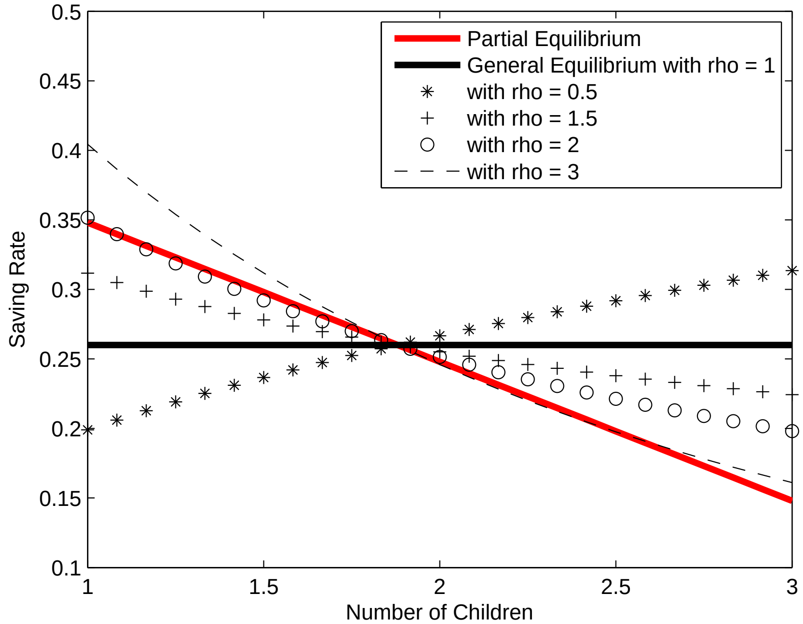


Figure 3: PE vs GE for $\lambda = \frac{1}{3}$



ONLINE APPENDIX – NOT FOR PUBLICATION

A Data Appendix

The sample frame used in the RUMiC is the same as the one used in the National Bureau of Statistics (NBS) *Annual Urban Household Income and Expenditure Survey* (UHIES). Sample selection is based on several stratifications at the provincial, city, county, township, and neighborhood community levels. Households are randomly selected within each chosen neighborhood community. The UHIES covers all 31 provinces, whereas the UHIES sample households were drawn from nineteen cities in nine of the provinces of the UHIES sample.[36] This sampling frame typically miss migrant laborers. For our study, this is an advantage in that we can assume that urban households we observe in 2008 also had urban status when they had their first child.

The survey was conducted in March and April, 2008. In addition to general information (including fertility) for household members, the questionnaire also included the demographic characteristics, education, and employment situation of other family members who are not residing with the household head and spouse, including parents, children, and siblings.[37] This allows us to know the total fertility history and characteristics of adult children such as sex, age and marital status. In our study, total fertility is synonymous with the total number of living children. In our sample, the total number of living children is very similar to the total number of children ever born since infant mortality during the early 1970s was very low (Banister and Hill, 2004).

The information on household income and expenditure from the RUMiCI in

[36]The provinces included in the RUMiCI urban survey are: Shanghai, Guangdong, Jiangsu, Zhejiang, Henan, Anhui, Hubei, Sichuan, and Chongqing. The detailed list of cities can be found at http://rumici.anu.edu.au

[37]The questionnaires are available from http://rumici.anu.edu.au

China are directly recorded from the UHIES survey, a 0.01% sample of all urban households that record income and expenditure variables using a diary record. Specifically, households are required to record each item (disaggregated for hundreds of product categories) purchased and income received for each day for a full year (in our case it is for the year 2007). Enumerators visit sample households once or twice each month to review the records, assist the household with questions, and to take away the household records for data entry and the aggregation of the annual data at the local Statistical Bureau Office.

The UHIES data is the best available data on urban household economic variables. It is not publicly available, but has been used in several recent studies. The data also have several weaknesses, which has been thoroughly discussed in by past studies such as Han, Cramer, and Wahl (1997), Ravallion and Chen (1999) and Gibson, Huang, and Rozelle (2003). According to these studies, the quality of the household surveys are in general good and most of the problems are confined to rural surveys. However, there are problems in the urban surveys that could affect studies of savings. First, the indicators used for consumption and expenditure lack consistency over time (e.g. the categories for durable consumption changed quite dramatically during two decades of rapid economic growth). Second, the urban surveys do not fully account for food consumption because they do not account for meals consumed away from home, although this is accounted for in expenditures for food. Finally, the onerous task of recording a daily diary of income, consumption and expenditure makes it difficult to recruit certain households. The first problem should not affect our study as we only use one cross-section and focus on urban residents. The second problem could cause us to underestimate consumption. We address this by using data on expenditures, which have been shown by the studies we cite above to be more accurate for urban household surveys. There is little we can do to directly address the last problem except to keep it in mind when considering the external validity of our results. According to interviews with NBS statisticians and a detailed examination of income and expenditure distributions conducted by researchers in study of the income distribution and income taxation using the UHIES data, researchers concluded that the households that refuse to participate are typically the poorest and the richest households (Piketty and Qian, 2009). This makes it difficult to use the UHIES to study the extreme tails of the income distribution, but should not affect our study, which focuses on the mean household.

Another important fact to keep in mind when assessing the external validity of our estimates is that China is the only country in the world that uses such comprehensive twelve month expenditure records.[38] Gibson, Huang, and Rozelle (2003) found that extrapolating annual totals from expenditures using some months of the year caused sharp decreases in expenditure measures.[39] This means that measures of household savings in China – the difference between income and expenditure – are not directly comparable to measures of household savings from other countries. (Unlike expenditures, income data is collected in a similar fashion as many other countries). In other words, if the same statistical methods employed in most of the world were also employed in China, then Chinese savings rates will be higher than what they are in our data (or any savings data that is based off of the UHIES). This error in measurement of what will be the dependent variable in our analysis should not affect our estimates. However, it needs to be taken into account when comparing

[38]Surveys in many other countries observe households for a week, a fortnight, or a month, and estimates of income and consumption from these periods are annualized by multiplying by 52, 26, or 12. The length of the recall period typically depends on the category of consumption, with long reference periods used for costly and/or infrequently consumed items and short reference periods for frequently consumed and minor items that would be easily forgotten (ILO., 1980).

[39]They also found that such extrapolations sharply increased measures of inequality. This may be due to the fact that by using data from only a few months, random shocks to expenditures are given too much weight. Also, see Deaton (1997) for a detailed discussion of the statistical tradeoffs of different data collection methods.

mean savings rates in China with other countries. Specifically, one would need to know the correlation between household's expenditures with different months.

There are several household level surveys from China. The UHIES (1988 -) surveys contain high quality income and expenditure data, but do not report total fertility. The China Health and Nutritional Surveys (CHNS) urban sample is small. The China Household Income Project (CHIP) does not report complete fertility and has a very small urban sample. The China Health and Retirement Longitudinal Survey (CHARLS, 2008, 2011) contains similar information to our survey and in addition, report transfers. We use these data for our descriptive statistics. Once we apply our sample restrictions, the CHARLS and RUMiC provide similar sample sizes for our study. Unfortunately, we are unable to use the CHARLS for the regression analysis because many of the linking variables for the full wave surveys are not yet available.

In our data, total household income is the sum of incomes from labor, business, property, pension and retirement allowances and other social welfare benefits. Total expenditure is the sum of consumption expenditure (e.g. food; clothing; housing; family equipment; service; health; transpirations and communication; education; cultural and entertainment; other commodity and services), operational expenditure, property expenditure, social security expenditure (e.g. individually paid pension fund, individually paid public housing fund, individually paid health care fund, individually paid unemployment fund, and other social security).[40]

[40]Food expenditure is the sum of expenditure on the following categories: grain, wheat, and rice coarse grains; pork, beef, and mutton; edible vegetable oil, fresh vegetables, dried vegetables, poultry, meat, eggs, fish; sugar, cigarettes, liquor, fruit, wine, beer, fresh melons and fruits cake; and milk.

B Details on Further Generalizations

B.1 Endogenous Fertility Choice

We now extend the model of section 4.1 and allow parents to optimally decide how many children to raise. Children are an indivisible good, so that parents may choose $n_{i,t+1} \in N$, where N is the set of non negative integers. We also assume that parents have heterogenous costs of raising children, in order to have a non degenerate distribution of fertility choices, and we let the cost of raising children to be convex in the number of children itself. This assumption is necessary in order to have a unique optimal solution for each household. The parameter $\gamma > 1$ controls the degree of convexity. Last, fertility is constrained by a possibly binding constraint Λ. As an example, the relaxation of the one-child policy can be modeled in this context as an increase in Λ. The household problem now reads as

$$\max_{c_{i,t}^{Y}, c_{i,t+1}^{O}, n_{i,t+1} \in N} log\left(c_{i,t}^{Y}\right) + \beta log\left(c_{i,t+1}^{O}\right)$$

$$s.t.$$

$$c_{i,t}^{Y} + \frac{c_{i,t+1}^{O}}{1+r_{t+1}} \leq A_t w_t \left(1 - \tau - \theta_i n_{i,t+1}^{\gamma}\right) + \frac{A_{t+1} w_{t+1}}{1+r_{t+1}} \tau n_{i,t+1}$$

$$n_{i,t+1} \leq \Lambda$$

The optimal saving rate of the model is identical to the one of section 4.1, and is given by

$$s_{i,t} = \left[\frac{\beta}{1+\beta}\right]\left[\left(1 - \tau - \theta_i n_{i,t+1}\right) - \frac{\tau n_{i,t+1}}{\beta\left(1+r_{t+1}\right)}\left(\frac{A_{t+1} w_{t+1}}{A_t w_t}\right)\right].$$

The difference with the baseline model is that now the optimal number of children is endogenous. In order to describe household behavior is useful to first consider the latent number of children, $\tilde{n}_{i,t+1}$, that would be optimally chosen if household could have any real number of children. This is given by

$$\tilde{n}_{i,t+1} = \left[\left(\frac{A_{t+1} w_{t+1}}{A_t w_t} \right) \left(\frac{\tau}{\gamma (1 + r_{t+1})} \right) \left(\frac{1}{\theta_i} \right) - \tilde{\mu}_i \right]^{\frac{1}{\gamma - 1}}$$

where $\tilde{\mu}_i \geq 0$ is the rescaled multiplier on the constraint $n_{i,t+1} \leq \Lambda$. It is immediate to notice that, as long as the constraint is not binding, $\tilde{n}_{i,t+1}$ is strictly decreasing in θ_i. However, households cannot have a fraction of a child, so that true fertility, $n_{i,t+1}$, jumps discretely. In particular, it is easy to verify that for each value $n = \{1, 2, ..., \Lambda\}$ $\exists \theta_n, \theta_{n-1}$ such that if $\theta_i = \theta_n$ then $n_{i,t+1} = n$ and if $\theta_{n-1} \leq \theta_i < \theta_n$ then $n_{i,t+1} = n - 1$.

In order to understand the implications of this model for the partial equilibrium estimates on the relationship between savings and fertility, it is interesting to compare two households which are identical, but for the observed number of children. In particular let's assume that household 1 has $\theta_1 = \theta_n$ and household 2 has $\theta_2 = \theta_1 - \epsilon$, where ϵ is a very small number. Household 1 is going to have n children, while household 2 is going to have $n - 1$ children. We can then compare the saving rates of the two households: since ϵ is very small is immediate to see that $s_2 > s_1$: household 2 has less children and thus saves more. The model therefore is consistent with the partial equilibrium evidence that shows, comparing households that are identical but for the number of children, that fertility and savings display a negative relationship.

Let's now discuss the general equilibrium implications of the model for aggregate fertility changes. As an illustrative example, let's consider the effect on savings of an aggregate reduction in fertility as caused by a tightening of the fertility constraint. Within the model, we thus consider the effect on fertility of a decrease in Λ. The reduction in fertility is going to have the same general equilibrium effects on prices as in the baseline model of section 4.1. Specifically, the reduction in fertility is going to reduce the interest rate and, as long as the economy is out of steady state, increase the growth rate of wage. However, the effect of the decrease in Λ on households behavior is going to be different for different groups of households. In particular we need to distinguish between two different possibilities. The first type of households is represented by those that are constrained by the tightening of Λ. Those households are going to decrease their fertility, and for them the analysis is identical to the case with exogenous fertility reduction: the extent to which their saving rate is going to increase depends on the relative strength of the consumption and transfer channels and on the responses of prices. There is however a second type of households, namely those that are not constrained even after the tightening of Λ. Those households are going to increase fertility on average. This is easy to see from the fact that, keeping $\tilde{\mu}_i$ fixed at zero (since those households are not constrained the multiplier is zero), the latent number of children is going to increase due to fact that $\frac{w_{t+1}}{w_t}$ increases and $1 + r_{t+1}$ goes down. Hence, this second group of households is going to increase fertility and consequently reduce savings. As a consequence, the effect of the tightening of Λ on aggregate saving rate is further dampened by the general equilibrium effects on this second group of individuals, beyond what it is in the case with exogenous fertility.

B.2 Endogenous Investment in Human Capital

We extend the model of section 4.1 and allow parents to optimally invest in their children's human capital. We first consider the case in which in partial equilibrium there is no quantity-quality trade-off, so that nor the costs nor the benefits of investing in children human capital depend from the number of children itself. We model human capital as an increase in individual productivity. The wage income of an individual i at time t is thus given by $A_t w_t h_{i,t}$. Aggregate income is produced, as in the baseline case, with a Cobb-Douglas production, where labor is now calculated in efficiency unit, as standard in the human capital literature, so that $Y = K_t^{\alpha}(A_t h_t L_t)^{1-\alpha}$, where h_t is the average human capital of the working population. Due to the assumption of competitive markets, the interest rate is $1+r_t = \alpha k_t^{\alpha-1} h_t^{1-\alpha}$ and wage per efficiency unit is $w_t = (1-\alpha) h_t^{-\alpha} k_t^{\alpha}$. Parents may invest in the human capital, $h_{i,t+1}$, of their children paying a convex cost $A_t w_t h_{i,t+1}^{\gamma}$, where $\gamma > 1$. Parents are willing to invest in the human capital of their children in order to increase received transfers: if children have more human capital they earn more and thus transfer more to parents. The problem of a household thus read as follows:

$$\max_{c_{i,t}^{Y}, c_{i,t+1}^{o}, h_{i,t+1}} \log\left(c_{i,t}^{Y}\right) + \beta \log\left(c_{i,t+1}^{O}\right)$$

$$s.t.$$

$$c_{i,t}^{Y} + \frac{c_{i,t+1}^{O}}{1+r_{t+1}} \leq A_t w_t h_{i,t}\left(1 - \tau - \theta \frac{h_{i,t+1}^{\gamma}}{h_{i,t}} n_i\right) + \frac{A_{t+1} w_{t+1}}{1+r_{t+1}}\left(\tau h_{i,t+1} n_i\right)$$

Solving the first order conditions of the model we obtain an equation for optimal saving rate and human capital investments

$$s_{i,t} = \left[\frac{\beta}{1+\beta}\right]\left[\left(1 - \tau - \theta \frac{h_{i,t+1}^{\gamma}}{h_{i,t}} n_i\right) - \frac{\tau h_{i,t+1} n_i}{\beta\left(1+r_{t+1}\right)}\left(\frac{A_{t+1} w_{t+1}}{A_t w_t h_{i,t}}\right)\right], \tag{15}$$

$$h_{i,t+1} = \left[\frac{A_{t+1} w_{t+1} \tau}{\gamma A_t w_t \theta\left(1+r_{t+1}\right)}\right]^{\frac{1}{\gamma-1}}, \tag{16}$$

which shows us that, at the household level, optimal human capital does not depend on the number of children, but only on parameters that are identical across households, so that $h_{i,t+1} = h_{t+1}\ \forall i$. Next, we focus on steady states and substitute 16 into 15 to get

$$s_i = \left[\frac{\beta}{1+\beta}\right]\left[\left(1-\tau-\frac{(1+g)\,\tau n_i}{\gamma\,(1+r)}\right)-\frac{(1+g)\,\tau n_i}{\beta\,(1+r)}\right] \tag{17}$$

from which we see that, even in the presence of endogenous human capital investment, fertility and savings are negatively related at the household level.

Let's now solve for the general equilibrium. The law of motion of capital is given by

$$k_{t+1} = (1-\alpha)\,\frac{s_t}{(1+g)\,n_{t+1}} h_t^{1-\alpha} k_t^{\alpha}$$

so that in steady state

$$k^{\alpha-1} h^{1-\alpha} = \frac{n\,(1+g)}{s\,(1-\alpha)}$$

and hence, using the definition of the interest rate, we get that in steady state

$$1+r = \frac{n\,(1+g)\,\alpha}{s\,(1-\alpha)}.$$

Substituting the equilibrium interest rate into 17 and summing over all households yield a formula for the aggregate saving rate

$$s = \frac{\alpha\beta\gamma\,(1-\tau)}{\alpha\gamma\,(1+\beta)+\tau\,(1-\alpha)\,(\beta+\gamma)}$$

which is independent from aggregate fertility. As such, despite the fact that at household level fertility and savings are negatively related, aggregate fertility and aggregate savings are not related.

This result come straight from the equation 16 for human capital investment. At the household level, human capital investment does not depend on the number of children, but is decreasing in the interest rate. At the aggregate level, however, human capital investment is decreasing in fertility: an increase in fertility increases the interest rate which makes the returns from investing in children human capital smaller. A quantity-quality trade-off thus emerges in general equilibrium, due to the role of fertility on the interest rate. Due to the assumptions about the functions made in the model, the decrease in human capital investment exactly compensate the "expenditure channel" relationship between fertility and savings. The consequence is that in general equilibrium there is no relationship between fertility and savings.

Partial Equilibrium Quantity-Quality Trade-off Alternatively, we could consider the case in which a quantity-quality trade-off is present also in partial equilibrium. A partial equilibrium quantity-quality trade-off can be modeled as a cost of human capital investment that is increasing in the number of children, so that the cost of investing in children human capital is now given by $\zeta\left(n_{t+1}\right) A_t w_t h_{t+1}^{\gamma}$, where $\frac{\partial \zeta(n_{t+1})}{\partial n_{t+1}} > 0$. This would imply that households with more children invest less in the human capital of each one of them. The saving rate and optimal human capital are now given by

$$s_{i,t} = \left[\frac{\beta}{1+\beta}\right]\left[\left(1-\tau-\theta\frac{h_{i,t+1}^{\gamma}}{h_{i,t}}\zeta\left(n_i\right)n_i\right)-\frac{\tau h_{i,t+1}n_i}{\beta\left(1+r_{t+1}\right)}\left(\frac{A_{t+1}w_{t+1}}{A_t w_t h_{i,t}}\right)\right],$$

$$h_{i,t+1} = \left[\frac{A_{t+1}w_{t+1}\tau}{\gamma\zeta\left(n_i\right)A_t w_t \theta\left(1+r_{t+1}\right)}\right]^{\frac{1}{\gamma-1}}.$$

Substituting the optimal human capital into the saving rate, and focusing to a steady state in which the number of siblings of parents and children is identical[41], we obtain again 17, so that the presence of partial equilibrium quantity-quality trade-off does not change the results previously shown.

[41] This assumption implies that $h_{i,t} = h_{i,t+1}$, which is useless to simplify the algebra and have stark results. We can relax this assumption and show that the general equilibrium relationship between fertility and savings is muted up to a covariance term. These results are available upon requests.

B.3 Endogenous Transfers to Parents

We extend the model of section 4.1 and let transfers from children to parents to be an endogenous outcome. In order to do so, we develop the model along the lines of Boldrin and Jones (2002)[42]. Individuals value their own consumption and the wealth of their parents. Every individual thus solves

$$\max_{c_t^Y, c_{t+1}^m, \tau_t} \log\left(c_t^Y\right) + \beta \log\left(c_{t+1}^O\right) + \delta \log\left(e_{t-1}^Y\right)$$

$$s.t.$$

$$c_{i,t}^Y + \frac{c_{i,t+1}^O}{1+r_{t+1}} \leq A_t w_t \left(1 - \tau_t\left(\tilde{n}_i\right) - \theta n_i\right) + \frac{A_{t+1} w_{t+1}}{1+r_{t+1}} \left(\tau_{t+1}\left(n_i\right) n_i\right)$$

$$e_{i,t-1}^Y \leq A_{t-1} w_{t-1} + \frac{A_t w_t}{1+r_t} \left(\tau_t\left(n_i\right) + \tilde{\tau}_t\left(n_i\right)\left(\tilde{n}_i - 1\right)\right).$$

The previous notation applies. Also notice that when deciding how much money to transfer to parents, individuals take as given the number of their siblings, $\tilde{n}_i$, and the transfer of their siblings, $\tilde{\tau}_t\left(n_t\right)$. We focus on a symmetric solution, so that in equilibrium $\tau\left(n\right) = \tilde{\tau}\left(n\right)$.

Solving the first order conditions of the model, we obtain the usual equation for optimal saving rate and an additional equation that comes from solving for the optimal transfer rate

[42] The Boldrin and Jones (2002) setting is slightly different than ours. They use a utility function of the form

$$U = \log\left(c_t^Y\right) + \beta \log\left(c_{t+1}^O\right) + \delta \log\left(c_t^O\right)$$

such that children value the consumption of their parents when parents are old, rather than parents well-being over the whole life. This assumption implies that parents have a strategic incentive not to save in the first period because savings crowd out transfers from children. We introduce the assumption that children care about the total wealth of the parents in such a way as to abstract from parents strategic behavior in savings. Conceptually, we are assuming that children have the ability to commit to a level of transfer that is independent from parents behavior in the first period, but depends only on the income of parents and macro economic condition.

$$s_{i,t} = \left[\frac{\beta}{1+\beta}\right]\left[(1-\tau_t(\tilde{n}_i) - \theta n_i) - \frac{\tau_{t+1}(n_i)\, n_i}{\beta(1+r_{t+1})}\left(\frac{A_{t+1}w_{t+1}}{A_t w_t}\right)\right], \tag{18}$$

$$c_t^Y = \frac{1}{\delta} e_{t-1}^Y (1+r_t). \tag{19}$$

Using 18, 19, and the budget constraints, we solve for the optimal transfer rate as a function of the number of siblings and children

$$\tau_t(n_i, \tilde{n}_i) = \frac{\left(\frac{1}{1+\beta}\right)\left[\frac{(1-\theta n_i)}{1+\beta} + \frac{A_{t+1}w_{t+1}}{A_t w_t}\frac{(\tau_{t+1}(n_i)n_i)}{(1+r_{t+1})(1+\beta)}\right] - \frac{1}{\delta}(1+r_t)\frac{A_{t-1}w_{t-1}}{A_t w_t}}{\frac{1}{1+\beta} + \frac{\tilde{n}_i}{\delta}}$$

The analysis of the optimal transfer rate is informative about the model implications for the partial and general equilibrium relationships between savings and fertility. The presence of endogenous transfer rate does not change the partial equilibrium relationship between fertility and savings[43], which is still given by the usual equation 18. In general equilibrium instead, the interest rate has now two effects on savings: (i) a wealth effect through the change in the value of transfers, which was present also in the model with exogenous transfer rate; (ii) a change in the transfer rate from each child. Both effects (i) and (ii) go in the same direction, so that general equilibrium forces are larger in the model with endogenous transfer rate. As an example, let's consider the foreseeable effects of the relaxation of the One Child Policy. The increase in aggregate fertility puts an upward pressure on the interest rate. The increase in the interest rate decreases the total value of transfers, so that parents save more with respect to the partial equilibrium prediction. This mechanism is identical to the corresponding effect in the model with exogenous transfer rate. But in addition, the increase in the interest rate implies that each child trans-

[43]The model implies that household level savings and fertility are negatively related, as long as we restrict the parameter set to obtain the natural assumption that households with more children receive more transfers.

fers less, because parents value future transfers less, which reduces the total amount of transfers and thus again increases savings. This additional channel means that the general equilibrium effect of an aggregate increase in fertility predicted by the model with endogenous transfer rate is smaller than the one predicted by the model with exogenous transfer rate.

To sum up, this analysis showed that if we believe that children transfer to parents as a result of altruistic behavior, then the general equilibrium relationship between fertility and savings is weaker than if we assume the transfer rate to be exogenous.

C Alternative Calibrations

In the main calibration exercise we have used point estimates from Table 4. However, as already discussed, two of three coefficients of interest are not precisely estimated, and are in fact not significant, with p-values of respectively 0.23 and 0.24. For this reason, in this section we perform a robustness exercise to understand the implications of our model for different sets of parameters. We allow the three coefficients of interest, namely the coefficient on the number of kids (ϕ henceforth), the coefficient on the interaction between the number of kids and the first born being a male (κ), and the coefficient on the first born being a male (ξ), to take one of three possible values: (i) the baseline value, which is simply the point estimates as shown in the Table 4; (ii) the baseline value minus its standard deviation multiplied by one third; and (iii) the baseline value plus its standard deviation multiplied by one third. The choice of one third is motivated by the fact that we want the transfer rate (τ) implied by the model to be positive, and the maximum values of ϕ, κ, and ξ that are consistent with τ being positive are in fact $\phi + \frac{1}{3}\sigma_{\phi}$, $\kappa + \frac{1}{3}\sigma_{\kappa}$, and $\xi + \frac{1}{3}\sigma_{\xi}$. We have 3 values for each of the three coefficients of interest, hence we have 27 possible combinations. For each of them we find the primitive parameters, τ, θ

and β, such that the model generates an average saving rate of 0.26 and matches the three coefficients of interest. We then use the calibrated model to perform the counterfactual exercises of increasing fertility by 1 child both in general and in partial equilibrium. We show that for almost all possible combinations of parameters the general and partial equilibrium effects of fertility on savings are very different. We now describe the results in more details.

For brevity, we focus on our preferred estimates, the one with $\rho = 1$ and $\lambda = 0$. In Table 7 we report the calibrated transfer rate (τ) for each triple of coefficients. Each matrix corresponds to one value for the coefficient on the first born being a male (ξ), each row to one value for the coefficient on the number of kids (ϕ), and each column to one value for the coefficient on the interaction between the number of kids and the first born being a male (κ). In Table 8 we report the calibrated consumption per child (θ). In Table 9 we report the percentage of the partial equilibrium effect on savings that is still present in general equilibrium. The table shows that for almost all combinations of coefficients the difference between partial and general equilibrium effects are sizable. The only exception is the case in which both ξ and κ take a high value. The reason is intuitive. When ξ and κ are high, the difference in saving rates between households with only one son and households with only one daughter is very small. This difference identifies the transfer rate, which is the driver of the general equilibrium effects. Indeed when ξ andκ are high the transfer rate is almost identical to zero, which implies that the relationship between fertility and savings is purely driven by the consumption channel. And, as shown in the paper, the consumption channel is identical in partial and general equilibrium.

Table 7: Transfer Rate

(a) Low ξ

	Low κ	Baseline κ	High κ
Low ϕ	25.02%	15.70%	10.91%
Baseline ϕ	21.87%	14.84%	10.46%
High ϕ	20.02%	14.10%	10.06%

(b) Baseline ξ

	Low κ	Baseline κ	High κ
Low ϕ	13.51%	8.81%	5.29%
Baseline ϕ	12.76%	8.44%	5.11%
High ϕ	12.12%	8.11%	4.94%

(c) High ξ

	Low κ	Baseline κ	High κ
Low ϕ	6.47%	3.05%	0.22%
Baseline ϕ	6.20%	2.95%	0.23%
High ϕ	5.96%	2.85%	0.24%

Table 8: Consumption per Child

(a) Low ξ

	Low κ	Baseline κ	High κ
Low ϕ	4.42%	5.30%	6.00%
Baseline ϕ	3.25%	3.79%	4.27%
High ϕ	1.83%	2.10%	2.37%

(b) Baseline ξ

	Low κ	Baseline κ	High κ
Low ϕ	10.47%	11.77%	13.09%
Baseline ϕ	9.24%	10.36%	11.54%
High ϕ	7.88%	8.83%	9.84%

(c) High ξ

	Low κ	Baseline κ	High κ
Low ϕ	17.12%	18.92%	20.85%
Baseline ϕ	15.97%	17.66%	19.49%
High ϕ	14.71%	16.28%	18.01%

Table 9: Ratio between the GE and PE effect on saving

(a) Low ξ

	Low κ	Baseline κ	High κ
Low ϕ	13.84%	17.12%	21.80%
Baseline ϕ	10.17%	12.74%	16.50%
High ϕ	5.86%	7.45%	9.85%

(b) Baseline ξ

	Low κ	Baseline κ	High κ
Low ϕ	32.68%	24.89%	52.76%
Baseline ϕ	29.81%	37.77%	49.56%
High ϕ	26.44%	33.99%	45.54%

(c) High ξ

	Low κ	Baseline κ	High κ
Low ϕ	58.21%	74.00%	97.31%
Baseline ϕ	56.43%	72.62%	97.13%
High ϕ	54.34%	70.96%	96.90%