

**Cost Channel, Interest Rate Pass-Through and Optimal Monetary
Policy under Zero Lower Bound**

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Abstract

Cost channel introduces trade-off between inflation rate and output gap. Unlike the canonical New Keynesian DSGE model, optimal monetary policy cannot set both inflation rate and output gap simultaneously to zero under a demand shock. Using a perfect foresight New Keynesian model with cost channel, this paper analyzes the optimal discretionary monetary policy under Zero Lower Bound (ZLB) for varying degree of interest rate pass-through. We find (i) exit date from ZLB becomes endogenous due to the trade-off between output gap and inflation introduced by the cost channel; (ii) presence of cost channel delays the exit from ZLB compared to models without cost channel; and (iii) exit date rises monotonically with the magnitude of demand shock and degree of interest rate pass-through.

Keywords: New-Keynesian Model, Inflation Target, Liquidity Trap

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1 Introduction

The recent financial crisis has witnessed negative natural rate of interest due to large adverse demand shock for a number of developed countries. This forces some of the major central banks, e.g. Federal Reserve Bank, Bank of England, Bank of Japan and the European Central Bank to reduce their target interest rates to (near) zero. The monetary authority loses its ability to lower the nominal interest rate further to stimulate economy when the economy hits the ZLB. Recent crisis has already rekindled interest to analyze the optimal policy under ZLB. Jung, et. al. (2005), Eggertson and Woodford (2003), Adam and Billi (2006), Nakov (2008) and Chattopadhyay and Daniel (2016) has analyzed the optimal policy under ZLB using a canonical New Keynesian DSGE model. The papers show that commitment outperforms discretion by promising future boom and inflation and also by delaying exit from ZLB. Assuming no uncertainty, this paper analyzes the optimal discretionary policy under ZLB using a New Keynesian DSGE model with cost channel.

The canonical New Keynesian DSGE model is the workhorse of modern monetary theory and policy. However, the model has been criticized for various properties with does not match the stylized facts (see, Mankiw and Reis, 2002). The property of “Divine Coincidence” is one of the reasons for disregarding the traditional New Keynesian DSGE model. According to the “Divine Coincidence” phenomenon, demand shock does not introduce any trade-off between inflation rate and output gap. As a result, monetary authority can simultaneously set inflation rate and output gap to zero by equating nominal interest rate with natural rate of interest (see, Woodford, 2002). Therefore, welfare loss both under discretion and commitment becomes zero.

Ravenna and Walsh (2006) introduces a cost channel in an otherwise canonical New Keynesian DSGE model by assuming that firms borrow from financial intermediaries at an interest rate to pay for their wage bill. The paper analyzed the optimal policy under discretion and commitment. They show that cost channel introduces the required trade-off to the system that breaks the “Divine Coincidence”. However, Ravenna and Walsh (2006) does not incorporate the ZLB of nominal interest rate in their analysis of optimal policy. This paper analyzes the optimal policy under discretion in a New Keynesian DSGE model with cost channel where ZLB constraint is binding. For simplicity we have not introduced any uncertainty in our analysis. We obtain some interesting results. First, while the exit date from ZLB is exogenous in a model without cost channel, exit date is determined endogenously under discretion when cost channel is present; second, presence of cost channel delays the exit; and third, exit date rises monotonically with magnitude

of demand shock and degree of interest rate pass-through.

The canonical New Keynesian DSGE model is usually concerned with demand side effect of monetary policy transmission. It is a standard belief that a contractionary monetary should lead to a reduction in prices through an adverse effect on aggregate demand. However, the majority of empirical literature is plagued by the problem of the so called price puzzle. “Price puzzle” is an occurrence where a contractionary monetary policy shocks identified with an increase in short-term interest rates, leads to a persistent rise in price level. Presence of cost channel is a prime suspect of “price puzzle” (see, Barth and Ramey, 2001).

The cost channel of monetary policy transmission can be explained by the phenomenon where the marginal cost of production of firms increase with a rise in interest rates. This is due to the fact that firms borrow from financial intermediaries to make payments for their factors of productions. Hence a higher interest rate increases firm’s borrowing costs which in turn, raise their marginal costs and ultimately leads to higher price level and inflation.

There are many other empirical studies establishing the importance of supply side effect of cost channel. However, there is no unanimous agreement about the presence of cost channel in the literature. Barth and Ramey (2001) provide evidence of cost channel by measuring a VAR model using aggregate and industry level data for the period 1959-2000. They argue that monetary policy shock should be treated as a supply side shock as the characteristics of impulse responses due to a monetary policy shock is similar to a productivity shock, which on the other hand, is very different from the impulse responses obtained from various other demand shocks.

In a standard NKM setting with real and nominal rigidities and a fraction of firms borrowing money to pay for their wage bill, Rabanal (2007) shows that demand side effect dominates the supply side effects of monetary transmission. Hence the cost channel fails to generate a price puzzle. Later, a related study by Henzel et al. (2009) using a minimum distance approach in contrast to Rabanal’s (2007) Bayesian technique, estimated a New Keynesian DSGE model for the Euro area. Henzel et al. (2009) showed that though the cost channel fails to generate a price puzzle for the Euro area, however, its presence explain the initial hump in prices due to a monetary policy tightening.

Chowdhury et. al. (2006) have estimated a hybrid version of New Keynesian Phillips curve with cost channel through GMM and have shown a significant presence of cost channel in majority of G-7 countries like Canada, France, Italy, UK and US. Ravenna and Walsh (2005) have also established a significant presence of cost channel by estimating

Phillips curve for the period 1960-2001 for the US.

Tillman (2008) also shows evidence of cost channel for US, UK and Euro area by estimating a forward looking hybrid Phillips Curve for each country using quarterly data for the time period 1960-2004. A year later, Tillman (2009) use a rolling window GMM estimate to assess time varying nature of cost channel. He assessed and compared the time varying effect of cost channel of monetary policy transmission through different business cycles, policy regimes and different structure of financial intermediations over the time for the US. The paper finds importance of cost channel in pre-Volker era and post Volker-Greenspan era.

Although there is myriad empirical literature to assess the presence, importance and characteristic of cost channel, theoretical works related to the optimal policy under cost channel are very limited. Ravenna and Walsh (2006) shows that unlike the traditional New Keynesian DSGE model, cost channel introduces a trade-off between output gap and inflation rate. As a result both output gap and inflation rate cannot be at the same time set equal to zero either under discretion or commitment. Chowdhury et. al. (2006) introduces credit market imperfection into the model of Ravenna and Walsh (2006). Araujo (2009) analyze the optimal policy in the model of Chowdhury et. al.(2009). The paper shows how the variance of output gap and inflation rate changes with the degree of credit market imperfection.

However, neither Ravenna and Walsh (2006) nor Araujo (2009) incorporates the ZLB of interest rate into their optimal policy analysis. We have analyzed the optimal policy under discretion without uncertainty using the model of Araujo (2009) when ZLB constraint is binding. Presence of trade-off between output gap and inflation under cost channel produces results including a delayed exit date which is determined endogenously under discretion and the exit date rises monotonically with the magnitude of demand shock and the degree of interest rate pass-through.

2 New Keynesian Model with Cost Channel

We use the New Keynesian model with cost channel proposed by Ravenna and Walsh (2006). The demand side of the model is the log linearized version of individual Euler equation around zero inflation steady state. The aggregate demand or the expectational IS equation is given by,

$$y_t = E_t(y_{t+1}) - \sigma [i_t - i - E_t(\pi_{t+1})] - u_t \quad (1)$$

Ravenna and Walsh (2006) assumes that, firms borrow at an interest rate to pay wages to labor. As a result, the marginal cost of firm depends on both output gap and interest rate. The presence of interest rate in the marginal cost of firms captures the required cost channel. Ravenna and Walsh (2006) assumes there is no financial market imperfection. As a result, they have lending and borrowing rates are equal to each other.

Araujo (2009) introduces degree of interest rate pass through in the model of Ravenna and Walsh (2006). Araujo (2009) assumes, $\iota_t^l = (1 + \psi) \iota_t = \delta \iota_t$, where, $\iota_t = i_t - i$ is the deviation of lending rate from long-run real interest rate and $\iota_t^l = i_t^l - i$ is deviation of deposit rate from long-run real interest rate. $\psi \in [0, 1)$ measures the degree of interest rate pass through. When there is no interest rate pass through we have, $\psi = 0$ and $\delta = 1$. In this case, the model of Araujo (2009) is identical with Ravenna and Walsh (2006). The log linearized intertemporal profit maximization of firm around zero inflation steady with Calvo price setting and labor market equilibrium gives the New Keynesian Phillips curve with cost channel as,

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(\sigma^{-1} + \eta) y_t + \kappa \delta (i_t - i) \quad (2)$$

Note, we have our standard New Keynesian Phillips curve without cost channel when $\delta = 0$. Following, Ravenna and Walsh (2005) we assume, even if rise in interest rate increase inflation rate, the negative effect of output dominates so that, tighter monetary policy actually reduces inflation rate. Therefore, we restrict, $\delta \in [0, \frac{\sigma^{-1} + \eta}{\sigma^{-1}})$.¹

In these equations y_t denotes the output gap; inflation (π_t) is the deviation about a long-run value of zero; i_t denotes the nominal interest rate (deposit rate), with a long-run equilibrium value of $i = r = \frac{1-\beta}{\beta}$, with r defined as the long-run real interest rate; σ represents the intertemporal elasticity of substitution with $\sigma \geq 1$, κ represents the degree of price stickiness; $\beta \in (0, 1)$ denotes the discount factor; η denotes the inverse of elasticity of Frisch labor supply and u_t represents the combination of shocks associated with preferences, technology, fiscal policy, etc.

¹Note, unit rise in nominal interest rate reduces output gap by σ unit through IS equation, which in turn reduces inflation by $\frac{(\sigma^{-1} + \eta)}{\sigma^{-1}}$ through by Phillips curve. However, unit rises in nominal interest rate increase inflation by $\kappa \delta$ unit by Phillips curve. Therefore, negative output effect dominates when, $\delta < \frac{(\sigma^{-1} + \eta)}{\sigma^{-1}}$. Also see, section 4.1.2 for restriction on δ .

3 Optimal Monetary Policy

The model is completed with determination of the nominal interest rate. Nominal interest is determined by optimal policy where the central bank chooses values for the time paths of inflation and the output gap to minimize the loss function,

$$L_t = \frac{1}{2} E_1 \sum_{t=1}^{\infty} \beta^{t-1} (\pi_t^2 + \lambda y_t^2), \quad \lambda \in [0, \infty). \quad (3)$$

where, λ is the relative weight on output gap relative to inflation rate. Ravenna and Walsh (2005) has derived loss function as a second order linear approximation to the utility function of the representative agent when equilibrium inflation is zero. Ravenna and Walsh (2005) assumes government expenditure is a fraction of output gap. Here, our objective is to analyze the optimal monetary policy under ZLB. As a result, we assume that there is no government expenditure. The loss function derived by Ravenna and Walsh (2005) boils down to the loss function given in equation (3) when there is no government expenditure.

The objective of the monetary authority is to minimize the loss function given in equation (3) subject to the expectational IS equation, (1), New Keynesian Phillips curve, equation (2) and the feasibility constraint, $i_t \geq 0$.

Ravenna and Walsh (2005) and Araujo (2009) has already analyzed the optimal monetary policy when $i_t > 0$. The major difference as highlighted by them between optimal policy with and without cost channel is the trade-off between inflation and output gap. Without cost channel, demand shock does not introduce any trade-off between inflation and output gap. This happens since demand shock causes both inflation and output gap to move together in same direction without cost channel. As a result, when the only shock is to the Euler equation, it is optimal to set $\pi_t = y_t = 0$ when cost channel is absent. Given these values, it is straightforward to show that the optimal value for the nominal interest rate without cost channel is,

$$i_t = i - \sigma^{-1} u_t = r_t^n, \quad (4)$$

where, r_t^n is defined as the natural rate of interest and $i_t > 0$ when ZLB is not binding.

According to equation (4), a reduction in the demand for current output (rise in u_t) reduces the natural interest rate and should be offset by a reduction in the nominal interest rate. The nominal interest rate should remain lower as long as demand and the natural rate are lower. An interest rate which fully offsets demand shocks keeps both inflation

and the output gap at their target values of zero. A nominal interest rate, set according to equation (4), is compatible with the target values of zero for inflation and the output gap. However, there is a trade-off between inflation and output gap when ZLB is binding (see, Jung, et. al., 2005, Adam and Billi, 2007, Eggertsson and Woodford, 2003 Nakov, 2008 and Chattopadhyay and Daniel, 2015).

However, cost channel introduces a trade-off between inflation rate and output gap even when ZLB is not binding as shown in Ravenna and Walsh (2006). The reason of trade-off in the presence of cost channel is the direct effect of interest rate on inflation through Phillips curve, which is absent without cost channel.² This paper analyzes the optimal policy under discretion in the presence of cost channel for various degree of interest rate pass-through when ZLB is binding. The analysis is done without uncertainty and will be extended to uncertainty later.³

4 Optimal Policy in ZLB under Discretion

The objective of the monetary authority is to minimize the loss function given in equation (3) subject to the expectational IS equation given in equation (1), New Keynesian Phillips curve with cost channel given in equation (2) and the feasibility constraint, $i_t \geq 0$. Moreover, we assume that there is no uncertainty and demand shock follows the following deterministic dynamics,

$$u_t = \rho^{t-1} u_1$$

The Lagrangian of the problem is,

$$\mathcal{L} = \left\{ \begin{array}{l} -\frac{1}{2} [\pi_t^2 + \lambda y_t^2] - \phi_{1,t} [\sigma (i_t - \pi_{t+1} - r_t^n) - y_{t+1} + y_t] \\ -\phi_{2,t} [\pi_t - \kappa (\sigma^{-1} + \eta) y_t - \kappa \delta (i_t - i) - \beta \pi_{t+1}] \\ + \phi_{3,t} i_t \end{array} \right\}$$

²Note, if interest rate rises due to adverse demand shock output gap falls. The reduction in output gap reduces inflation through Phillips curve. However, the rise in interest rate increases inflation rate directly through Phillips curve. This introduces a trade-off between inflation and output gap as shown in Ravenna and Walsh (2005).

³Chattopadhyay and Daniel (2015) shows that, uncertainty merely changes the exit date from ZLB. The exit can be earlier or delayed in the presence of uncertainty. A favorable demand shock causes early exit than an unfavorable demand shock. The paper also shows that, post-exit dynamics remains unchanged even if we introduce uncertainty. As a result, the major results remain unaltered even if we introduce uncertainty.

The First Order Conditions are,

$$\frac{\partial L}{\partial \pi_t} = -\pi_t - \phi_{2,t} = 0 \quad (5)$$

$$\frac{\partial L}{\partial y_t} = -\lambda y_t - \phi_{1,t} + \kappa (\sigma^{-1} + \eta) \phi_{2,t} = 0 \quad (6)$$

$$\frac{\partial L}{\partial \phi_{1,t}} = \sigma (i_t - \pi_{t+1} - r_t^n) - y_{t+1} + y_t = 0 \quad (7)$$

$$\frac{\partial L}{\partial \phi_{2,t}} = \pi_t - \kappa (\sigma^{-1} + \eta) y_t - \kappa \delta (i_t - i) - \beta \pi_{t+1} = 0 \quad (8)$$

$$\frac{\partial L}{\partial i_t} = -\sigma \phi_{1,t} + \kappa \delta \phi_{2,t} + \phi_{3,t} = 0 \quad (9)$$

$$\phi_{3,t} \frac{\partial L}{\partial \phi_{3,t}} = \phi_{3,t} i_t = 0, \phi_{3,t} \geq 0, i_t \geq 0 \text{ with complementary slackness} \quad (10)$$

Equation (5) and (6) gives,

$$\phi_{1,t} = -(\lambda y_t + \kappa (\sigma^{-1} + \eta) \pi_t) \quad (11)$$

Again, from equation (9) and (10) $\phi_{3,t} \geq 0$ implies,

$$\sigma \phi_{1,t} - \kappa \delta \phi_{2,t} \geq 0 \quad (12)$$

Note, exit depends only on $\phi_{1,t}$ when cost channel is absent ($\delta = 0$). To determine exit time under cost channel define, $Q_t = \kappa (\delta - \sigma (\sigma^{-1} + \eta)) \pi_t - \sigma \lambda y_t$. Then, equation (6), (11) and (12) gives,

$$\begin{aligned} i_t &= 0 \text{ till } Q_t > 0 \\ &> 0, \text{ O.W.} \end{aligned} \quad (13)$$

Higher credit market imperfection increases Q_t and delays exit. After exit, i_t is determined by expectational IS schedule, equation (1).

4.1 Model Solution under Discretion

We will solve the model without uncertainty. The entire solution has two parts. We assume that economy is in liquidity trap for $t = 1, 2, 3, \dots, T$ and out of liquidity trap from $t = T + 1$ onwards. This assumption means, that once out of trap, economy never comes back to liquidity trap again. We will first solve for $t = 1, 2, 3, \dots, T$ when economy is in

liquidity trap. Then we will solve for $t = T + 1$ onwards when economy is out of liquidity trap.

4.1.1 Periods $t = 1, 2, \dots, T$

For $t \leq T$, the value for the nominal interest rate is zero. Write equations (1) and (2) with $i_t = 0$ as,

$$Z_{t+1} = C + AZ_t - ar_t^n \quad (14)$$

where,

$$C = \begin{bmatrix} -\frac{\sigma\kappa\delta i}{\beta} \\ \frac{\kappa\delta i}{\beta} \end{bmatrix}, a = \begin{bmatrix} \sigma \\ 0 \end{bmatrix}, Z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix},$$

$$A = \begin{bmatrix} 1 + \frac{\sigma\kappa(\sigma^{-1} + \eta)}{\beta} & -\frac{\sigma}{\beta} \\ -\frac{\kappa(\sigma^{-1} + \eta)}{\beta} & \frac{1}{\beta} \end{bmatrix}$$

A forward looking solution of equation (14) yields

$$Z_t = \Gamma_t + \Omega_t \quad (15)$$

where,

$$\Gamma_t = \sum_{k=t}^T A^{-(k-t+1)} ar_k^n$$

$$\Omega_t = \left(A^{-(T-t+1)} Z_{T+1} - \sum_{k=t}^T A^{-(k-t+1)} C \right)$$

Equation (15) implies that values for deviations of inflation and the output gap prior to exit from the ZLB depend on their expected values on the date of exit from the ZLB. The promise to exit the ZLB with positive values for inflation and the output gap stimulate the economy while at the ZLB. However, while economy exits with zero inflation and output gap without cost channel, cost channel introduces trade-off between inflation and output gap. Hence, economy exits with either with positive output gap or inflation. Additionally, postponement of the exit date with a larger value for T , stimulates since the coefficients in the $A^{-(T-t+1)}$ matrix are increasing in T . The exit date and Z_{T+1} are unique when we have no uncertainty. Next we solve the model post exit from liquidity trap.

4.1.2 Periods $t = T + 1, T + 2, \dots$

In the period in which the economy exits the ZLB, the nominal interest rate becomes positive and remains positive. This implies, $\phi_{3,t} = 0$ for $t = T + 1, T + 2, \dots$. Equation (5), (6) and (9) with $\phi_{3,t} = 0$ gives,

$$\pi_t = -\chi y_t \quad (16)$$

where,

$$\chi = \frac{\lambda}{\kappa [\sigma^{-1} (1 - \delta) + \eta]}$$

Note, we need $\chi > 0$ to have trade-off between output gap and inflation rate. The post-exit inflation rate and output gap is zero without cost channel ($\delta = 0$). However, the post exit time path of inflation and output gap is obtained by solving (7), (8) and (16) simultaneously in the presence of cost channel, $\left(\delta \in [1, \frac{\sigma^{-1} + \eta}{\sigma^{-1}})\right)$. Eliminating nominal interest rate from (7), (8) gives,

$$y_{t+1} + \sigma \left(1 + \frac{\beta}{\kappa \delta}\right) \pi_{t+1} = \left[1 - \frac{\sigma^{-1} + \eta}{\sigma^{-1} \delta}\right] y_t + \frac{\sigma}{\kappa \delta} \pi_t + u_t \quad (17)$$

When, $\delta \in [1, \frac{\sigma^{-1} + \eta}{\sigma^{-1}})$, equation (16) is negatively sloped and equation (17) is positively sloped.

$$y_{t+1} = \mu y_t - \frac{u_t}{\sigma \chi \left(1 + \frac{\beta}{\kappa \delta}\right) - 1} \quad (18)$$

where,

$$\mu = \frac{\frac{\sigma}{\kappa \delta} (\chi + \kappa (\sigma^{-1} + \eta)) - 1}{\sigma \chi \left(1 + \frac{\beta}{\kappa \delta}\right) - 1}$$

Note, we need $\mu > 1$, so that we can solve equation (18) forward to obtain sunspot free bounded equilibrium.⁴. Calculating μ for different value of $\delta \in [1, \frac{\sigma^{-1} + \eta}{\sigma^{-1}})$ gives a threshold $\delta^* > 0$, such that, $\chi > 0$ but $\mu < 1$ when $\delta > \delta^*$. As a result, we restrict $\delta \in [1, \delta^*]$ to ensure equilibrium determinacy.

Forward solution of equation (18) gives,

$$y_t = \frac{u_t}{\left[\sigma \chi \left(1 + \frac{\beta}{\kappa \delta}\right) - 1\right] (\mu - \rho)}, \text{ for } t = T + 1, T + 2, \dots \quad (19)$$

Substituting, equation (19) to equation (16) gives optimal inflation. Figure 1 describes

⁴Note, if we introduce uncertainty, output gap becomes a jump variable. As a result, we need the root, $\mu > 1$ to get a sunspot free unique and bounded equilibrium.

the post-exit optimal response of output gap and inflation rate to adverse demand shock under discretion. Note, since $\delta \in [1, \delta^*]$, equation (16) is negatively sloped and equation (17) is positively sloped. Equation (16) is denoted by AA and equation (17) is denoted by BB in Figure 1. An adverse demand shock shifts BB curve down to BB_1 . This causes post-exit output gap to rise to y_1 and inflation rate to fall to π_1 .

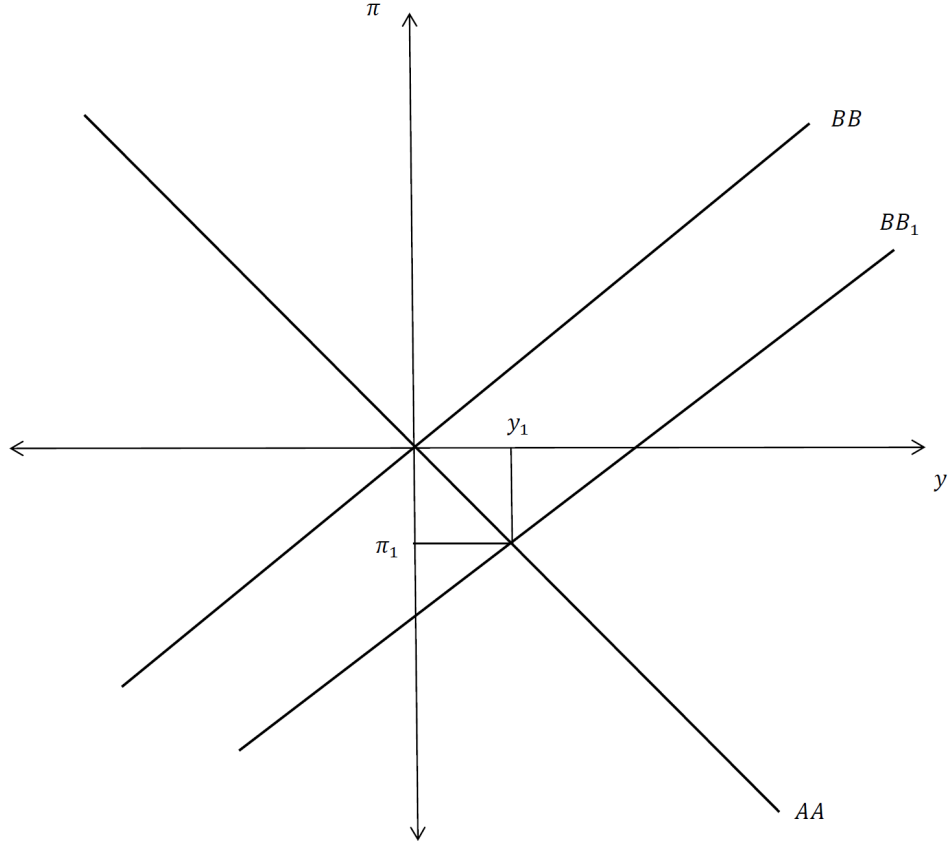


Figure 1: Post-exit Optimal Response to Adverse Demand Shock under Discretion

However, impact of δ on post-exit output gap is not monotonic. We see that, δ has threshold. When δ is below the threshold, μ falls at a faster rate and dominates the rise in $[\sigma\chi(1 + \frac{\beta}{\kappa\delta}) - 1]$. As result, we see higher post-exit fluctuation in output gap. However, $[\sigma\chi(1 + \frac{\beta}{\kappa\delta}) - 1]$ rises rapidly when δ goes beyond the threshold, causing lower fluctuation in output gap. On the other hand, inflation falls monotonically as δ

risers because χ rises monotonically with δ . In terms of Figure 1, higher δ makes the AA curve steeper and BB curve flatter.

Post-exit output gap and inflation gives,

$$Z_{T+1} = \begin{bmatrix} y_{T+1} \\ \pi_{T+1} \end{bmatrix} \quad (20)$$

Next, we calculate pre-exit time path for output gap and inflation rate given the terminal condition Z_{T+1} numerically from (15) for different values of T . The exit time under discretion is determined using equation (13). Once exit time is calculated, post-exit time path of output gap and inflation is calculated from equation (19) and equation (16) respectively. The pre-exit nominal interest rate is set to zero and the post-exit nominal interest rate is calculated from expectational IS equation. Note, while the exit time is completely exogenous without cost channel, exit time in the presence of cost channel is endogenous due to non-zero output gap and inflation produced by the post-exit endogenous trade-off between output gap and inflation rate.

4.1.3 No Cost Channel under Discretion

It is worth mentioning here that, post-exit output gap and inflation without cost channel are both zero and hence, pre-exit output gap and inflation rate is determined by,

$$Z_t^* = \Gamma_t$$

As a result, the difference between cost channel and no cost channel is given by,

$$Z_t - Z_t^* = \Omega_t$$

Therefore, the cost channel imparts less stimulus to output gap when its element in $\Omega_t < 0$ and vice-versa. The same applies to inflation rate as well.

Moreover, the exit date without cost channel is exogenous and determined entirely by the time path of natural rate of interest (as post-exit value of output gap and inflation are zero). Nominal interest rate (determined by expectational IS equation) remains zero as long as natural rate of interest is zero and becomes positive as soon as natural rate of interest becomes positive.

5 Calibration and Impulse Response

We illustrate the base line impulse response of output gap, inflation rate, nominal interest rate and real interest rate using following parameterization.

Table 1: Parameter Description			
Parameter	Description	Value	Source
σ	Logarithmic Preference	1	Adam and Billi (2006)
β	Discount Factor	0.99	Standard
κ	Response of Inflation to Output Gap in Phillips Curve	0.028	Adam and Billi (2006)
η	Inverse of Slope of Frisch Labor Supply	1	Walsh and Ravenna (2005)
λ	Relative Weight on Output Gap in Loss Function	0.0074	Adam and Billi (2006)

Discount factor, $\beta = 0.99$ implies long run real interest rate, $i = \beta^{-1} - 1 = 0.0101$. $\kappa = 0.028$ implies price is highly sticky with only 16% of firm can choose their price optimally each period. The slope of the Phillips curve without cost channel in Adam and Billi (2006) is $\kappa = 0.056$. The slope of Phillips curve in Ravenna and Walsh (2006) is $\kappa(\sigma^{-1} + \eta)$. I set, $\kappa = 0.028$ such that, $\kappa(\sigma^{-1} + \eta) = 0.056$ so that I can identify the impact of cost channel only.⁵ Moreover, we get $\delta^* = 1.8$ given the the parameterization so that $\chi > 0$ and $\mu > 1$. All values are expressed at quarterly rates.

Moreover, we need demand shock to be large enough to send the economy into liquidity trap. We also need liquidity trap to persists for a considerable period of time, Hence we set,

$$u_1 = 0.024, \rho = 0.9$$

Figure 2 gives optimal time path of output gap, inflation, nominal interest rate and real interest rate for different degree of credit market imperfection (δ). Table 2 reports the difference in optimal time path for cost channel relative to no cost channel under

⁵Note, lower price stickiness increase the value of κ . Cochrane (2014) shows that impact of recession due to ZLB is higher without cost as κ rises. This is true even when the cost channel is present. Since, our objective is to analyze the impact of cost channel on optimal policy when ZLB is binding, we have chosen a κ so that we get a reasonable fluctuation that matches data. Varying κ simply varies the fluctuations keeping the core result unchanged.

discretion.

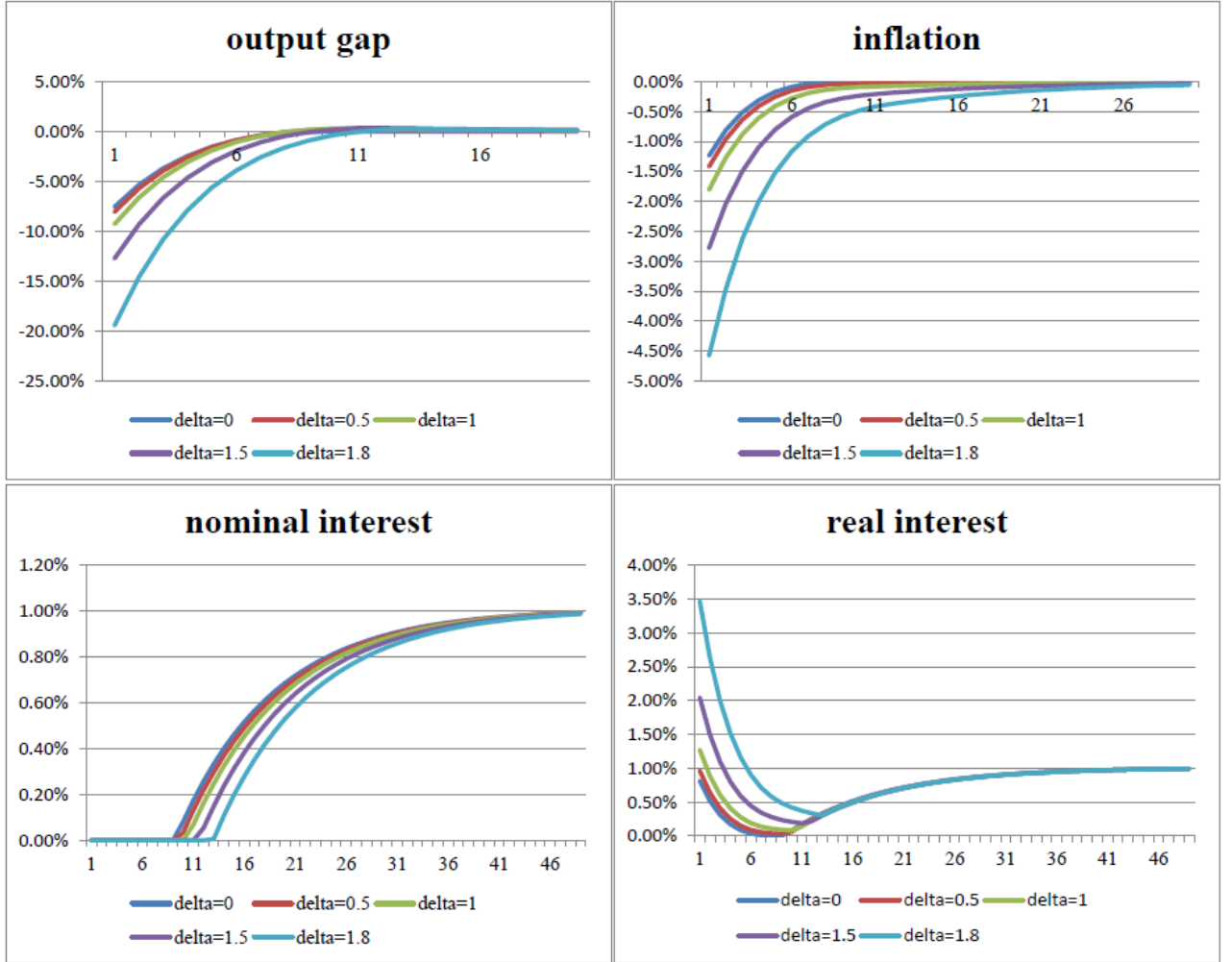


Figure 2: Optimal Time Path under Discretion for Different Degree of Interest Rate Pass-Through

We see in Figure 2 that large adverse demand shock puts economy into ZLB and causes recession and deflation. However, while post-exit output gap becomes positive and gradually converges to zero economy suffers from deflation for the entire time period. Output fluctuates more than inflation rate for any given δ , since coefficient associated with output gap is less than one in Phillips curve.

Table 2: Cost Channel and No Cost Channel under Discretion		
δ	T	Loss Relative to No Cost Channel
0	9	1
0.5	9	1.33
1	10	2.28
1.5	11	5.67
1.8	12	16.44

Again, deflation causes real interest rate to rise, which is consistent with recession as shown in Figure 2. Figure 2 also shows that, fluctuation of both output gap and inflation rate rises with δ for any given demand shock. As a result, welfare loss relative to no cost channel as shown in Table 2 is also rising with δ . This happens due to the presences of cost channel in Phillips curve. Note, higher δ reduces inflation through Phillips curve when ZLB is binding ($i_t = 0$). Higher deflation causes real interest rate to rise and higher reduction in output gap for a given demand shock.

Higher credit market imperfection also delays exit as shown in Table 2. Equation (13) which shows that Q_t rises with δ which explains the delayed exit with rising δ . Delayed exit with higher δ is also evident in time path of nominal interest rate in Figure 2.⁶ We see in Table 2 that, exit from ZLB is soonest when $\delta = 0$ and latest when $\delta = 1.8$ Table 2 also shows that, no cost channel gives the minimum time period required to exit from ZLB. Most importantly, contrary to without cost channel, exit date with cost channel is endogenous even under discretion due to post exit trade-off between output gap and inflation rate

6 Conclusions

Many studies have established the importance of supply side effects of cost channel. The cost channel of monetary policy transmission can be explained by the phenomenon where the marginal cost of production of firms increase with a rise in interest rates, which in turn, leads to higher price level. Hence, it is crucial to acknowledge both the

⁶Note, one eigenvalue of the coefficient matrix A is greater than one and other less than one. Hence A^{-1} rises with T imparts more stimulus to the system. We would have seen more welfare loss than reported in Table 2 (due to higher fluctuation in output gap and inflation) if exit date does not rise with δ .

demand side and supply side channels of monetary policy transmission as they seem to have collaborative effects on macro economy with the final effect (on prices and output) depending on the relative strength of the channels. The studies done so far in the literature have analyzed the optimal policy in presence of cost channel in a New Keynesian DSGE model. This paper extends the study of the optimal policy under discretion with cost channel and credit market imperfections when ZLB constraint is binding. Introduction of trade-off between output gap and inflation due to presence of cost channel produces the following results for an economy which is at the ZLB. First, exit date is determined endogenously under discretion when cost channel is present; second, presence of cost channel delays the exit; and third, exit date rises monotonically with magnitude of demand shock and degree of interest rate pass-through.

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