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Stakeholders, Bargaining and Strikes

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ABSTRACT

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We study bilateral bargaining problems with interested third parties, the stakeholders that enjoy benefits upon a bilateral agreement. We explore the strategic implications of this third party involvement. Our main finding is that the potential willingness of the stakeholder to make contributions to promote agreement may be the source of severe inefficiency. However, and more surprisingly, for a wide range of parameter values this outcome is better for the stakeholder than if he enters bargaining directly. Our results lend support to the tendency towards decentralisation of pay bargaining in the public sector in Europe.

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1 Introduction

In Marcel Pagnol's movie "La femme du boulanger"[1], the baker's wife runs away with her lover. But then, the baker bakes bread which his wife sells, so that without her the baker's enterprise is worthless. So, the baker goes on strike, sending a clear message to the village: no wife, no bread. The village goes up in arm, but there is no convincing the baker. The only option left to the villagers is to go on the hunt for the baker's wife, and bring her back to return harmony (and bread) to the French village.

This amusing story captures effectively the fact that conflict affecting certain services, whether publicly or privately provided, does concern, directly or indirectly, the interests of many parties. Bilateral bargaining in an area of public interest therefore has an impact on third parties, *stakeholders*, who are interested in the resolution of the conflict, yet unable to impose an agreement upon the contending bargainers[2]. In this paper we analyse the effect that stakeholders have on the bargaining outcome.

The mere fact that the interests affected by the conflict extend well beyond the two participants is what defines its "public interest" attribute. It is reasonable to assume that in many cases the public interest will have much wider consequences than the issue at stake in the bargaining. For instance, the disruption of essential services like public transport, hospitals, the fire service and the electricity, gas and water industries has a substantial impact on the population at large. Correspondingly, the government's stake in bilateral conflicts which are of public concern is of some consequence. That is, one can postulate that the government's stake can be quantified as 'greater' than the issue bargained over. This immediately presents a potential for exploitations from the two contenders: as long as the stakeholder has more to lose from disagreement, bargainers should succeed in extracting some resources from the stakeholder. Indeed, especially during the '70s and the '80s, Europe was hit by a wave of general strikes in which workers managed to win (mainly salary) concessions from the government of the day[3]. The following two decades have seen an effort of governments across Europe towards a greater flexibilisation of

[1] See [21], based on [11].

[2] Note that this feature distinguishes our model from the literature on bargaining with arbitration or mediation, in which a third party (the mediator) derives no utility from an agreement (e.g. [3], [16], [18], [22] and [27]).

[3] For instance, in England strikes affected most of the services of public interest, from transport (notably the dockers strikes in 1972 and 1980), the energy sector (water and gas services), and the National Health service (see [17]). In Italy industrial disputes touched mainly the transport sector (see e.g. [1]). For an hystorical account of industrial disputes across Europe in the '70s and '80s see [15] and [4].

employment laws which apply to employment in services of public interest, so as to "harmonise" it with the private sector[4]. These changes in the legislation regulating industrial relations, which in the mains have been directed at weakening the power of trade unions[5], have generally tended towards a decentralisation of bargaining in the public sector[6]. The effect has been to transform what would have been in essence a bilateral relationship into trilateral negotiations between management and union in the shadow of possible state intervention[7]. Consequently, the sheer possibility that the stakeholder (i.e. the government) may intervene in negotiations creates the potential for delays, in the hope to pressurise the government into conceding extra resources. So why wouldn't the stakeholder with the power to do so change the "rules of the game" to a more efficient negotiating framework[8]?

These are the type of issues we address in this paper. More precisely, we explore bilateral bargaining explicitly accounting for the presence of stakeholders. We model such bargaining problems as non-cooperative games with three players: two players, *the bargainers*, have the ability to reach an agreement; the third player, a stakeholder, can only take (limited) actions that condition the nature of the bilateral bargaining.

We start by looking at the strategic incentives in a simple bargaining model with perfect and complete information. Our main finding confirms our original suspicion that the presence of a stakeholder generates delays: stakeholders are usually willing to make contributions to pro-

[4]For instance in France on the one hand services which were traditionally in the public sector are now no longer publicly provided (e.g. social security services and supplementary pension schemes, which are managed by private companies). On the other hand in services of public interest which are privately provided the employment law is based onto the private employment law. Services which are of public interest but privately provided differ however in some important respect (e.g. they are not subject to collective bargaining).

[5]See for instance [5], [6] and [7].

[6]See [8] and [14].

[7]Canada is another case in point, as conciliators and mediators can be appointed by the government to help resolve management-union disputes. See [12].

[8]Note that it is very difficult for a stakeholder (i.e. the government) to limit its involvment by committing to some maximum amount of resources to bestow in order to avert stalemate in negotiations, and then let the bargainers to "fight it out". Besides the potential political cost of adopting this type of stance, it would be a non credible commitment, as the stakeholder has all of its stake to lose while the two bargainers haggle. Indeed, in real negotiations this has hardly been the case. For instance even in the UK, were, especially following the surge to power of Margaret Thatcher in 1979, legislation has been most effective in weakening trade union powers, the spending limits self-imposed by successive governments have been broken in order to honour previous pay commitments. A case in point is the Fire service threatened strike of 1980, which was averted by awarding a 18.8% rise, breaking the Conservative Government's guidlines for the public sector. See [19].

mote agreement, but this willingness may backfire and become the source of severe inefficiency. However, and more surprisingly, for a wide range of parameter values *this outcome is better for the stakeholder than a situation in which he bargains directly with the union.*

The intuition for this result is straightforward. Delays are harmful for all of the agents involved in negotiations; however, in trilateral negotiations the management and the stakeholder can join forces and secure as a coalition an ex ante expected payoff which is greater than what they would otherwise obtain as a single negotiator in bilateral bargaining with the union.

These results concern the situation where all parties are perfectly informed about the overall resources available. In practice, however, negotiators seldom have an accurate assessment of the means at the stakeholder's disposal. Thus in the second part of the paper we analyse the effects of uncertainty by building a simple model where bargainers are unsure about the stakeholder's stake. Note that besides being more realistic, this type of framework can account for situations where the stakeholder is genuinely *super partes*, or even antagonistic[9] to one or both of the other negotiators. We find that, not surprisingly, uncertainty does not remove inefficient (i.e. delayed) equilibria. However, as long as the stake is not too great, there are also equilibria in which the two litigants reach an efficient agreement with positive probability. This class of equilibria are driven by the expectation of the two bargainers that the stakeholder would not intervene in case of a stalemate, which makes it worthwhile for the bargainers to get to an agreement quickly. Consequently, it is optimal for the stakeholder to dither, thereby pressurising the two bargainers into reaching a speedy conclusion.

The general features of the situation that we model fit many contingencies beyond industrial relations, and the framework of analysis that we propose is general enough to model stake holders as "interested' parties, which may still be genuinely neutral. For example, arbitrating and mediating efforts by neutral third countries to promote peace settlements to end armed confrontations are possibly motivated by the assessment that more is at stake than the welfare of the actors directly involved in the conflict.

The rest of the paper is structured as follows. In the next section we introduce a benchmark model where all agents have claims on both the stakeholder and the bargainers surpluses. In section 3 we build a model of public sector bargaining. Section 4 analyses cases where both bargainers wish to capture some of the (uncertain) surplus available to the stakeholder. Finally

[9] This can be the case for instance when the government (the *indirect* employer) oversees negotiations between a union and a local government (the *direct* employer) which is controlled by the opposition party.

section 5 concludes.

2 Symmetric model - the benchmark

As a prelude to our public sector bargaining model, in this section we introduce a symmetric natural model for bilateral bargaining in the presence of a stakeholder. In this context any of the agents involved (be it a bargainer or the stakeholder) can make proposals over both the division of the surplus in the bargain *and* handouts from the stakeholder. The complete symmetry of this setup makes it unsuitable to capture the sort of strategic situation we wish to model. However it is useful in order to understand the effect of the strategic forces introduced by the presence of a stakeholder in bilateral bargaining.

Two agents (indexed 1 and 2) bargain over sharing some surplus, normalised to unity. They can make additional claims from a stakeholder s, who can make available extra resources up to the amount S. Negotiations proceed over a (potentially infinite) number of rounds. Each agent can be randomly selected to make a proposal, with equal probability $p_i = \frac{1}{3}$, where $i = 1, 2, s$. A proposal consists of a division of the surplus and a contribution vector of resources requested from the stakeholder. If both responding agents agree, then negotiations end and the agreement is implemented. If instead at least one agent rejects the proposal, then negotiations move to the following round, when again one agent is randomly selected to make a proposal, and so on. Agents discount utility over time by a factor δ_i. Consequently an agreement reached at time t on surplus vector $x = (x_1, x_2)$ and contribution vector $c = (c_1, c_2)$ yields utility $u_i(x, c, t) = \delta_i^t (x_i + c_i)$ to bargainer i and $u_g(x, c, t) = \delta_s^t \left(S - \sum_{i=1,2} c_i\right)$ to the stakeholder.

Strategies and Subgame Perfect Equilibria (SPE) are defined in the standard way. A *stationary strategy profile* is a time-independent plan of action for each i at each subgame in which i is called to move. Stationary strategy profiles are fully characterized by a vector specifying actions and acceptance thresholds (whichever applicable) for each player at each state of the game[10]. A *Stationary Subgame Perfect Equilibrium* (SSPE) is a stationary profile of strategies that constitutes an SPE. An *Immediate Agreement Equilibrium* (IAE) is a SSPE that yields agreement at $t = 0$, regardless of which player takes the first action. A *Delayed Agreement Equilibrium* (DAE) is an SSPE where rejection occurs with positive probability at states of the game that occur with positive probability.

[10]The states of the game at t are the (possibly empty) sequences of moves that have occurred in the t'th bargaining round.

Our first result establishes that there is a stationary equilibrium where agreement is reached immediately and where the stakeholder releases to the bargainers part, though not all, of his own resources.

Proposition 1 (IAE) *There exist* $\underline{S}_s$ *such that if* $S \geq \underline{S}_s$ *a family of IAE exists. For all parameter configurations such that an IAE exists the ex ante equilibrium payoff is* $\widehat{V}_i = \frac{(1-\delta_j)(1-\delta_s)(1+S)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)}$ *for each bargainer* i *and for the stakeholder* $\widehat{V}_s = \frac{(1-\delta_1)(1-\delta_2)(1+S)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)}$. *Equilibrium strategies supporting the above equilibrium are as follows:*

- Bargainer i:
- *Proposes* $\left(x^i, c^i\right)$ *such that* $x_i^i = \widehat{x}_i^i$, $c_i^i + c_j^i = \widehat{\gamma}$ *and* $c_i^i \in \left[\underline{b}_i, \overline{b}^i\right]$;
- *accepts any share* $\geq 1 - \widehat{x}_j^j$ *from Bargainer* j *with contribution vector* c^j *such that* $c_i^j + c_j^j = \widehat{\gamma}$ *and* $c_j^j \in \left[\underline{b}_i, \overline{b}^i\right]$;
- *accepts any proposal yielding a payoff* $\geq \delta_i \widehat{V}_i$ *from the stakeholder with contribution vector* c^s *such that* $c_1^s + c_2^s = \widehat{\gamma}^s$;
- Stakeholder:
- *proposes* (x^s, c^s) *with* $x_i^s + c_i^s = \delta_i \widehat{V}_i$ *and* $c_1^s + c_2^s = \widehat{\gamma}^s$;
- *concedes to bargainer* i *when the latter asks for contributions* c^i *such that* $c_i^i + c_j^i = \widehat{\gamma}$.

where

$$\widehat{x}_i^i = \frac{(1-\delta_j)(1-\delta_s)(3-2\delta_i)(1+S)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} - c_i^i$$
$$\widehat{\gamma}^s = \frac{((1-\delta_s)(\delta_1+\delta_2-2\delta_1\delta_2))S-(3-2\delta_s)(1-\delta_1)(1-\delta_2)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)}$$
$$\widehat{\gamma} = \frac{(1-\delta_s)(3-2\delta_2-2\delta_1+\delta_1\delta_2)S-\delta_s(1-\delta_1-\delta_2+\delta_1\delta_2)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)}$$
$$\underline{b_i} = \frac{(1-\delta_s)(3-2\delta_i)(1-\delta_j)S-(1-\delta_i)(\delta_s+\delta_j-2\delta_s\delta_j)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} \leq c_i^i \leq \frac{(1-\delta_s)(3-2\delta_i)(1-\delta_j)(S+1)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} = \overline{b_i}$$

Proof. Standard. See Appendix.

We now show that there can be no equilibria where agreement is delayed with positive probability. This is established in the following proposition:

Proposition 2 (DAE) *There can be no DAE.*

Proof. See Appendix.

To understand the above result, note that since we are dealing with stationary strategies, the only way to obtain delayed agreement is if in equilibrium some agent's proposal is always rejected. The consequence is that such an agent looses all the bargaining power deriving from the possibility of him being first mover. On the other hand, all of the agents "pay" the cost of a rejection by moving negotiations to the next round. This is however what destroys this as a possible equilibrium, as the agent whose offer is always rejected can "bank" on these costs and make an offer which makes him better off and that responders have an incentive to accept.

Consider first a candidate equilibrium where it is the stakeholder's proposal to be rejected always. In this case the bargainers can coordinate and extract the stakeholder's surplus completely, by insisting on a complete handout each time they make a proposal. The stakeholder would have no incentive to refuse such a proposal, as a rejection would trigger either a subgame where the stakeholder is a proposer, in which case no agreement follows; or one in which one of the bargainers makes the same offer the stakeholder has just rejected.

However, precisely for this reason, the above cannot be an equilibrium, for the stakeholder has an incentive to make an offer that would be accepted: for instance, he could offer immediately the same surplus division as a bargainer would do in the next round, and give out all of his stake bar a small amount. Similar arguments apply to the candidate equilibrium where a bargainer's offer is always rejected.

2.0.1 A remark on non-stationary equilibria.

Uniqueness of stationary equilibria does not rule out the existence of other non-stationary equilibria [11], and this is the case in the present model, too. However, non-stationary strategies are often very complex and, especially in order to support delays, punishment are needed which are often rather extreme, quite unlike what one is more likely to observe in practice[12]. As an example, below we provide a non stationary strategy profile which (given suitable restrictions on parameter values) supports an equilibrium with agreement in the second round:

- in the first round, each agent rejects all offers yielding less than the ex ante expected equilibrium payoff;
- in the second round:
  - bargainer i claims $\frac{2}{3}$ of the surplus (leaving $\frac{1}{3}$ to the opponent, which is accepted) and ask for no contributions from the stakeholder;
  - the stakeholder proposes $x = \left(\frac{1}{2}, \frac{1}{2}\right)$ and $c = \left(\frac{1}{6}, \frac{1}{6}\right)$, which are accepted;

[11] See eg. section 3.13 in [20].

[12] Complexity arguments can be used to justify stationary strategies in bargaining games. See for instance [25].

- bargainer i accepts any share $\geq \frac{1}{3}$ ($\geq \frac{1}{2}$) and any contribution ≥ 0 ($\geq \frac{1}{6}$) from bargainer j (from the stakeholder);
- the stakeholder concedes to any request of a contribution such that $c_1 + c_2 \leq \frac{1}{3}$

- Deviations are punished by reverting play to the following strategy profile: following a deviation by either the stakeholder or bargainer i, then in the punishment phase bargainer j proposes $x_i = 0$ and $x_j = 1$, with $c_i = 0$ and $c_j = \frac{1}{3}S$; bargainer i proposes $x_i = 0$ and $x_j = 1$, with $c_i = 0$ and $c_j = \frac{1}{3}S$; the stakeholder proposes $x_i = 0$ and $x_j = 1$, with $c_i = 0$ and $c_j = 0$. Note that in this case $\delta_s V_s = \frac{7}{9}\delta_s S$ and $\delta_j V_j = \frac{1}{9}\delta_j (9 + 2S)$.

Verification that these strategies constitute an equilibrium is relegated to the appendix.

In the equilibrium exhibited above, agreement is delayed just one period, so that it was possible to keep the punishment strategies relatively simple. Even so, the equilibrium described above is far from being straightforward. One appealing feature is that in the punishment profile proposed above, essentially what happens is that the stakeholder and one bargainer coalesce in order to punish the deviant bargainer. Being the one with more resources, the stakeholder is absolutely necessary to avoid the deviators from succeeding in "bribing" the other agents. On the other hand, because of this central role the stakeholder manages to retain a great deal of his stake. But is it necessary to rely on complex strategy profiles and extreme punishments, seldom observed in practice, to highlight the incentives that agents may have to collude at the expense of another agent? Indeed, one may ask the more general question regarding the circumstances which make it possible and profitable for two sides to join forces in these negotiations. This is the focus of the next section.

3 Public Sector Bargaining

In the previous section we presented a completely symmetric model in which (in stationary strategies) no inefficiencies arise: here the stakeholder is a true facilitator, as bargainers manage to extract part of the stake from him, and agreement is always reached immediately.

However, as the discussion in the introduction shows, real world negotiations in the presence of a stakeholder are seldom completely symmetric, as not all actors have a right to dispose of *all* the resources which are the object of overall negotiations. For instance, a government can give a handout to one or both contenders in a management-union wage dispute, but cannot impose a wage settlement to the management[13]. That is, there is some sort of hierarchical structure which seem to govern negotiations. As we will show, it is this sort of asymmetries that introduce the potential for inefficiencies in negotiations, even in a setup of complete and perfect information.

One could object that since with perfect information bilateral negotiations always ensure an efficient outcome[14], there is no reason why a third party, the stakeholder, should get involved. Yet, the situation at hand may be one such that the stakeholder cannot get away from it, as it is often the case in services of public interest. More importantly, though, we show below that the stakeholder may be able to turn any inefficiencies to its own advantage.

Consider the following natural model of bilateral bargaining in the presence of a stakeholder. Let the management and the union (indexed by m and u, respectively) bargain over sharing some surplus, normalised to unity. They can make additional claims from the stakeholding government g, who can make available extra resources up to the amount S. Negotiations, which proceed over a (potentially infinite) number of rounds, follow a hierarchic protocol, in the sense that the union can never approach the government directly, but only after having reached a tentative agreement with the management on the division of the unitary surplus. On the other hand, the government can propose a contribution directly to the union, which then has to reach agreement with the management on the division of the unitary surplus. Finally, the management negotiates with the union only, and any agreement reached when the management is an initiator is final.

Specifically, each agent is randomly selected to make the first offer with probability p_i, where $i = u, m, g$ and $\sum p_i = 1$. In what follows we use superscripts to refer to the randomly selected first mover. A proposal by the union (which is selected to make the first proposal with probability p_u), consists of a tentative division of the surplus $(x^u, 1 - x^u)$ and a claim c^u from the government. A proposal by the government (selected as proposer with probability p_g) consists of a contribution c^g to the union; if this is accepted, then the management can put forward a division of the $(x^g, 1 - x^g)$, while a proposal by the management (selected as proposer

[13]For instance, in 2000 BMW secured a £152m aid package from the British government by threatening to transfer car production from the UK Rover plant in Longbridge to eastern Europe. In some countries, however, (e.g. Norway) the government can impose upper bounds on wages (i.e. "wage freezes"). See however footnote 8 above.

[14]This is the case in the 'standard' alternating offers bargaining model. However it is well known that inefficient equilibria can obtain in alternating offers bargaining models even with complete information, once the original extensive form ([24]) is modified. See for instance [13] and [2].

with probability p_m) consists simply of a surplus division $(x^m, 1-x^m)$. In other words, with probability p_m the government stays out of negotiations. An agreement is implemented only when all parties concerned have agreed to it. If instead at least one agent rejects the proposal, then negotiations move to the following round, when again one agent is randomly selected to make a proposal, and so on.

Agents discount utility over time by a factor δ_i. We let $\delta_u = \delta$, while both employers in the public sector share the same discount factor $\delta_g = \delta_m = \gamma$. An agreement reached at time t on surplus vector $x = (x_u, 1-x_u)$ and contribution c yields utility $u_u(x,c,t) = \delta^t(x_u + c)$ to the union, $u_m(x,c,t) = \gamma^t(1-x_u)$ to the management and $u_g(x,c,t) = \gamma^t(S-c)$ to the stakeholding government.

Our first result is that an IAE is not guaranteed to exists.

Proposition 3 *An IAE exists iff* $S \leq \frac{(1-\delta)(1-p_u\gamma)}{\delta(1-\gamma)p_u} \equiv \overline{S}$.

Proof. An IAE is fully characterized by a vector of actions $(x^u, x^m, x^g, c^g, c^u) \in [0,1]^3 \times [0,S]^2$ where x^i, $i = u, m, g$, is the surplus division proposals in subgames where the initiator is agent i, and c^j, $j = u, g$, is the contribution proposed by agent j.

Let v^i denote the ex-ante equilibrium payoff to agent $i = u, m, g$ in a given SSPE and observe that the following must hold in an IAE:

$$c^g + x^g = \delta v^u$$
$$x^m = \delta v^u$$
$$1 - x^u = \gamma v^m$$
$$S - c^u = \gamma v^g$$

so that each responding agent obtains the present discounted value of his ex-ante expected payoff, that is:. Note that the first two equations imply that $c^g + x^g = x^m$. Furthermore, at an equilibrium proposals have to be optimal, that is:

$$x^u + c^u \geq \delta v^u$$
$$S - c^g \geq \gamma v^g$$
$$1 - x^g \geq \gamma v^m$$

Observe, moreover, that the equilibrium payoffs must solve:

$$\begin{aligned} v^g &= p_m S + p_g(S - c^g) + p_u(S - c^u) = \\ &= (1 - p_u - p_g)S + p_g(S - c^g) + p_u \gamma v^g, \end{aligned}$$

$$\begin{aligned} v^m &= p_m(1-x^m)+p_g(1-x^g)+p_u(1-x^u) = \\ &= (1-p_u-p_g)(1-\delta v^u)+p_g(1-\delta v^u+c^g)+p_u\gamma v^m, \end{aligned}$$

and

$$\begin{aligned} v^u &= p_m x^m + p_g(c^g+x^g)+p_u(c^u+x^u) = \\ & (1-p_u)\delta v^u + p_u(c^u+x^u); \end{aligned}$$

yielding

$$\begin{gathered} v^m = \frac{(1-p_u)(1-\delta v^u)+p_g c^g}{1-p_u\gamma} \\ v^g = \frac{(1-p_u)S-p_g c^g}{1-p_u\gamma} \\ v^u = \frac{p_u(S+1-\gamma(v^g+v^m))}{1-(1-p_u)\delta} \end{gathered}$$

Substituting

$$(v^m+v^g)\,\gamma = \gamma\frac{(1-p_u)(1+S-\delta v^u)}{1-p_u\gamma}$$

into the expression for v^u we obtain that

$$v^u = \frac{p_u\left(S+1-\left(\gamma\frac{(1-p_u)(1+S-\delta v^u)}{1-p_u\gamma}\right)\right)}{1-(1-p_u)\delta} = p_u\frac{(S+1)(1-\gamma)}{1-\delta+p_u(\delta-\gamma)} \quad .$$

so that

$$(v^m+v^g) = \frac{(1-p_u)(1-\delta)}{1-\delta+p_u\delta-p_u\gamma}\,(S+1)$$

Consequently we can determine

$$x^m = \delta v^u = \delta p_u\frac{(S+1)(1-\gamma)}{1-\delta+p_u(\delta-\gamma)} = c^g + x^g \tag{1}$$

and observing that

$$1-x^u+S-c^u = \gamma\,(v^g+v^m)$$

we obtain

$$x^u+c^u = \frac{(1-\gamma)(1-\delta(1-p_u))}{1-\delta+p_u\delta-p_u\gamma}\,(S+1) \tag{2}$$

For these quantities to characterise an equilibrium we need $x^m \in [0,1]$, or[15]

$$S \le \frac{(1-\delta)(1-p_u\gamma)}{\delta(1-\gamma)p_u} \equiv \overline{S}$$

[15] Note that x^m is positive as long as the denominator is, that is $\gamma < \frac{1-\delta(1-p_u)}{p_u}$, which is always true as long as $\frac{1-\delta(1-p_u)}{p_u} \ge 1$.

Furthermore, it is easy to verify[16] that $x^u + c^u < S + 1$ and that the optimality of proposers' strategies holds[17].

Note that $S \leq \overline{S}$ is sufficient for the existence of an IAE, as we can construct a strategy profile with proposal vectors satisfying equations 1 and 2 which are an equilibrium. ■

Our next result is that when an IAE fails to exist, there are DAE. In DAE the initial actions (proposals) of m are rejected.

Proposition 4 *For* $S > \underline{S}_{p_g} = \frac{(1-\gamma)(1-\delta)}{(\delta p_u(1-\gamma)+\gamma p_g(1-\delta))}$ *there is a continuum of DAE; in all these this equilibria m's initial proposals is always rejected.*

Proof. Consider a stationary strategy profile where in subgames where m is the first mover m's initial proposal to the union is never accepted. One such strategy profile is fully characterized by a vector $(x^u, x^g, c^g, c^u) \in [0, S]^2 \times [0, 1]^2$ where x^i, $i = u, m, g$, is the surplus division proposals in subgames where the initiator is agent i, and c^j, $j = u, g$, is the contribution proposed by agent j.

Note that if (x^u, x^g, c^g, c^u) are the proposals in a DAE, then the ex-ante expected payoffs must solve

$$v^g = p_g(S - c^g) + (1 - p_g)\gamma v^g \Leftrightarrow v^g = \frac{p_g(S-c^g)}{1-\gamma+\gamma p_g}$$
$$v^m = p_g(1 - x^g) + (1 - p_g)\gamma v^m \Leftrightarrow v^m = \frac{p_g(1-x^g)}{1-\gamma+\gamma p_g}$$
$$v^u = (1 - p_u)\delta v^u + p_u(S + 1 - \gamma(v^g + v^m)) \Leftrightarrow v^u = p_u\frac{S+1-\gamma(v^g+v^m)}{1-\delta+\delta p_u}$$

from which

$$v^g + v^m = \frac{p_g(S-c^g)}{1-\gamma+\gamma p_g} + \frac{p_g(1-x^g)}{1-\gamma+\gamma p_g} = \frac{p_g(S+1-c^g-x^g)}{1-\gamma+\gamma p_g}$$

[16] This requires

$$\frac{(1-\gamma)(1-\delta(1-p_u))}{1-\delta+p_u\delta-p_u\gamma}(G+1) < G+1 \Leftrightarrow -\gamma(1-p_u)(1-\delta) < 0$$

which holds always.

[17] For this we need $x^u + c^u \geq \delta v^u$, which is equivalent to

$$(1-\delta)(1-\delta+p_u\delta-p_u\gamma) \geq 0$$

and

$$G + 1 - (c^g + x^g) \geq \gamma(v^m + v^g) = \gamma\frac{(1-p_u)(1-\delta)}{1-\delta+p_u\delta-p_u\gamma}(G+1)$$

which holds if

$$\frac{(1-\gamma)(1-\delta)}{1-\delta+p_u\delta-p_u\gamma} \geq 0$$

which is always true.

Furthermore (c^g, c^u, x^u, x^g) must satisfy

$$c^g + x^g = \delta v^u$$
$$S - c^u = \gamma v^g$$
$$1 - x^u = \gamma v^m$$

where the last two equations require

$$c^u + x^u = S + 1 - \gamma\left(v^g + v^m\right)$$

Substitution of $c^g + x^g = \delta v^u$ yields

$$v^g + v^m = \frac{p_g(S+1-\delta v^u)}{1-\gamma+\gamma p_g}$$

so that

$$v^u = \frac{p_u\left(S+1-\gamma\frac{p_g(S+1-\delta v^u)}{1-\gamma+\gamma p_g}\right)}{1-(1-p_u)\delta} \Leftrightarrow v^u = \frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}$$
$$v^g + v^m = \frac{p_g\left(S+1-\delta\frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}\right)}{1-\gamma+\gamma p_g} \Leftrightarrow v^g + v^m = \frac{p_g(S+1)(1-\delta)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}$$

Correspondingly we can determine

$$x^g + c^g = \delta\frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}$$
$$x^u + c^u = 1 + S - \gamma\left(v^g + v^m\right) = \frac{(1-\gamma)(1-\delta(1-p_u))}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}\left(S+1\right)$$

Now for (c^g, c^u, x^u, x^g) to be a DAE we must ensure that the other optimality conditions hold, that is we require:

$$x^u + c^u \geq \delta v^u$$
$$S - c^g \geq \gamma v^g$$
$$1 - \delta v^u \leq \gamma v^m$$

where the last inequality ensures that the management does not have a profitable deviation. It is easy to verify[18] that the first of these inequalities is satisfied. For the second inequality, using$1 - x^u = \gamma v^m$ we can re-write the requirement as

$$S - c^g + 1 - x^u \geq \gamma\left(v^g + v^m\right) \Leftrightarrow x^u + c^g \leq 1 + S - \gamma\left(v^g + v^m\right) = x^u + c^u$$

[18] One can substitute the corresponding equilibrium values in $x^u + c^u \geq \delta v^u$ to verify that this inequality is satisfied if $1 - \delta \geq 0$, which holds true always.

requiring simply

$$c^g \leq c^u$$

so that the government always makes a smaller concession when proposing rather than when approached by the union. Finally, for the third inequality we can use $S - c^u = \gamma v^g$ to express it as

$$1 + S - \delta v^u - c^u \leq \gamma \left(v^g + v^m\right)$$

to obtain

$$c^u \geq 1 + S - \delta \frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} - \gamma \frac{p_g(S+1)(1-\delta)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}$$

or

$$c^u \leq \frac{(1-\delta)(1-\gamma)(S+1)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} \equiv \overline{c^u}$$

Note that $\overline{c^u} < S$ if[19]

$$(1-\delta)(1-\gamma)(S+1) - S\left((1-\delta+\delta p_u)(1-\gamma) + \gamma p_g(1-\delta)\right) < 0$$

or

$$S > \frac{(1-\gamma)(1-\delta)}{(\delta p_u(1-\gamma)+\gamma p_g(1-\delta))} \equiv \underline{S}_{p_g}$$

Hence, provided $S > \underline{S}_{p_g}$, a DAE is characterised by a vector (c^g, c^u, x^u, x^g) with $c^g, c^u \in [0, S]$ and $x^u, x^g \in [0, 1]$ such that

$$\begin{aligned} c^g &\leq c^u \\ x^g + c^g &= \delta \frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} \\ x^u + c^u &= \frac{(1-\gamma)(1-\delta(1-p_u))}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)}(S+1) \end{aligned}$$

[19] A *sufficient* condition for existence of the DEA is that $1 - \delta v^u = 1 - \delta\overline{u} < 0$, that is

$$1 - \delta \frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} < 0 \Leftrightarrow \frac{(1-\delta)\left(1-\gamma+\gamma p_g\right) - S p_u \delta(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} < 0$$

which holds if

$$S > \frac{(1-\delta)\left(1-\gamma+\gamma p_g\right)}{p_u \delta(1-\gamma)} \equiv \underline{S}_1$$

Note that $\underline{S}_1 > \underline{S}_{p_g}$, since

$$\underline{S}_1 - \underline{S}_{p_g} = (1-\delta)\gamma p_g \frac{(1-\delta)(1-\gamma)+\delta p_u(1-\gamma)+\gamma p_g(1-\delta)}{\delta p_u(1-\gamma)\left(\delta p_u(1-\gamma)+\gamma p_g(1-\delta)\right)} > 0$$

■

The following is now immediate:

Corollary 5 *For each S, and p_u, there are values δ, γ such that in any SSPE all proposals by m are rejected, and the expected date of agreement is*

$$E(t) = (p_g + p_u)\sum_{k=0}^{\infty}(k+1)(1-p_g-p_u)^k = \frac{1}{p_g+p_u}$$

It is important to observe that there is an overlap between the range of government resources which guarantee existence of IAE and DAE. Indeed, $\underline{S}_{p_g} \leq \overline{S}$, since this holds if

$$\frac{(1-\gamma)(1-\delta)}{(\delta p_u(1-\gamma)+\gamma p_g(1-\delta))} \leq \frac{(1-\delta)(1-p_u\gamma)}{\delta(1-\gamma)p_u}$$

or

$$\gamma\left(p_u\left(1-\gamma\right)\delta\left(1-p_u\right)+p_g\left(1-\delta\right)\left(1-\gamma p_u\right)\right) \geq 0$$

which is always true. Consequently, there are parameter values such that multiple stationary equilibria can coexist. It seems consequential to check the welfare implications of the two class of equilibria. More precisely, we can compare the ex ante expected payoff to the public sector (management and government) and the union in the two equilibria.

Observe that the ex ante expected equilibrium payoff to the public sector is always higher in a IAE than in a DAE, since

$$\left(v^g+v^m\right)^{DAE} = \frac{p_g(S+1)(1-\delta)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} < \frac{(1-p_u)(1-\delta)}{1-\delta+p_u\delta-p_u\gamma}\left(S+1\right) = \left(v^m+v^g\right)^{IAE}$$

is true if

$$\begin{gathered} p_g\left(1-\delta+p_u\delta-p_u\gamma\right)-\left(1-p_u\right)\left((1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)\right)<0 \Leftrightarrow \\ -\left(1-\gamma\right)\left(1-\delta\left(1-p_u\right)\right)\left(1-p_u-p_g\right)<0 \end{gathered}$$

On the contrary, the ex ante expected payoff to the union is always greater under DAE than under IAE, since

$$\left(v^u\right)^{DAE} > \left(v^u\right)^{IAE}$$

can be rearranged as

$$\begin{gathered} \frac{p_u(S+1)(1-\gamma)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} > p_u\frac{(S+1)(1-\gamma)}{1-\delta+p_u(\delta-\gamma)} \Leftrightarrow \\ \gamma\left(1-\delta\right)\left(1-p_u-p_g\right)>0 \end{gathered}$$

which is always true. These results are easy to understand by noting that in any DAE there is never agreement in subgames when the government is not involved. Thus, in any equilibrium agreement the union anticipates that it is going to obtain some concessions, which pushes up the ex ante expected equilibrium payoff. On the other hand, the ex-ante equilibrium expected payoff for the public sector is affected adversely by the fact that the management's bargaining power is weakened as the management can only be a responder in equilibrium.

Finally, note that as the probability that the government makes an offer, p_g, goes to zero, the ex-ante expected payoffs for both sides (public sector and union) converge to the same values under DAE and IAE, and $\underline{S}_{p_g} \rightarrow \overline{S}$. If this is the case, though, DAE and IAE cannot coexist. Furthermore, in this case the expected time of agreement is pushed forward, to $\frac{1}{p_u}$.

In summary, then, IAEs always make the public sector better off with respect to DAE when they exist. However, if the stakes available to the government are large enough, immediate agreement cannot be supported in equilibrium, and delays occur.

3.1 Centralised versus decentralised bargaining

The discussion above begs the question of whether it would not be worth it for the public sector to centralise bargaining and act as a single agent at the outset. We now show that in spite of their inefficiency, there may be DAE that are strictly better for g and m than the unique and efficient SPE outcome of a bilateral (i.e. centralised) bargaining game over $S+1$.

Let us first recall the payoffs that prevail in a bilateral bargaining over $S+1$.

Remark 1 *A bilateral bargaining game between the union and a single agent p over $1+S$,where the public sector initiates the bargaining with probability p_p at each t has the unique SPE and yields payoffs:*

$$\begin{aligned} v^u &= \frac{(1-\gamma)(1-p_p)(S+1)}{1-\delta p_p - \gamma(1-p_p)} \text{ to } u \text{ and} \\ v^p &= \frac{(1-\delta)\, p_p\,(S+1)}{1-\delta p_p - \gamma(1-p_p)} \text{ to } p \end{aligned}$$

(It is straightforward that these values solve $v^u = \frac{1}{2}(S+1-\gamma v^p) + \frac{\delta}{2} v^u$ and $v^p = \frac{1}{2}(S+1-\delta v^u) + \frac{\gamma}{2} v^p$)).

A IAE of the game of public sector bargaining yields the public sector a higher payoff than under bilateral bargaining if and only if

$$(v^m + v^g) = \frac{(1-p_u)(1-\delta)}{1-\delta+p_u\delta - p_u\gamma}(S+1) > \frac{(1-\delta)p_p(S+1)}{1-\delta p_p - \gamma(1-p_p)} = v^p$$

that is if

$$\frac{(1-p_u)}{1-\delta+p_u\delta-p_u\gamma} > \frac{p_p}{1-\delta p_p-\gamma(1-p_p)}$$

which can be rearranged as[20]

$$(1-p_u)\left(1-\delta p_p-\gamma\left(1-p_p\right)\right) > p_p\left(1-\delta+p_u\delta-p_u\gamma\right) \Leftrightarrow$$
$$(1-\gamma)\left(\left(1-p_p\right)-p_u\right) > 0$$

which holds as long as

$$p_u < (1-p_p)$$

That is, for separation to be effective under IAE it is sufficient that the probability that the union gets to make the first offer in the decentralised game is lower than in the bargaining game. In other words, the public sector is better of in the decentralised game as long as its bargaining power as measured by the cumulative probability that it gets to make a proposal is higher than under bilateral bargaining[21].

A similar type of results obtains under DAE. Equilibria in this class are characterised by the fact that the management's offer is never accepted in equilibrium. Consequently, the crucial comparison is now going to be between the probability that the public sector makes an offer in bilateral bargaining and the probability that the government alone makes an offer under decentralised bargaining. Under DAE any equilibrium in this class yields the public sector a higher payoff than under bilateral bargaining if and only if

$$v^g + v^m = \frac{p_g(S+1)(1-\delta)}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} > \frac{(1-\delta)p_p(S+1)}{1-\delta p_p-\gamma(1-p_p)} = v^p \tag{3}$$

that is

$$\frac{p_g}{(1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)} > \frac{p_p}{1-\delta p_p-\gamma(1-p_p)}$$

[20] Note that both denominators are positive.

[21] For instance, it is easy to verify that if both under IAE and bilateral bargaining each agent can be selected as a proposer with equal probability, then IAE yields a higher ex-ante expected payoff than bilateral bargaining:

$$\frac{\left(1-\frac{1}{3}\right)(1-\delta)}{1-\delta+\frac{1}{3}\delta-\frac{1}{3}\gamma}(S+1) - \frac{(S+1)(1-\delta)}{2-\gamma-\delta} = (S+1)(1-\delta)\frac{(1-\gamma)}{(3-2\delta-\gamma)(2-\gamma-\delta)} > 0$$

Noting that the denominators are both positive it is enough to require

$$p_g\left(1-\delta p_p-\gamma\left(1-p_p\right)\right)>\left((1-\delta+\delta p_u)(1-\gamma)+\gamma p_g(1-\delta)\right)p_p \Leftrightarrow$$
$$(1-\gamma)\left(p_g\left(1-\delta p_p\right)-p_p\left(1-\delta\left(1-p_u\right)\right)\right)>0 \Rightarrow$$
$$\left(p_g\left(1-\delta p_p\right)-p_p\left(1-\delta\left(1-p_u\right)\right)\right)>0 \Leftrightarrow$$
$$\delta>\frac{p_p-p_g}{p_p(1-p_g-p_u)}=\underline{\delta}_{p_g p_u}$$

where

$$\underline{\delta}_{p_g p_u}\leq 1 \Leftrightarrow p_g>\frac{p_u p_p}{(1-p_p)}=\underline{p_g}$$

Note that the denominator in the expression defining $\underline{\delta}_{p_g p_u}$ is always positive. If $p_g > p_p$, then the numerator is negative, so that the restriction is always satisfied. If instead $p_g < p_p$, ceteris paribus the public sector is weaker under decentralised bargaining than under bilateral bargaining.. However, provided than the union is sufficiently patient (which increases its bargaining power in bilateral negotiations), the ex-ante expected payoff for the public sector is higher in the DAE than in bilateral bargaining.

The above considerations can be summarised in the following proposition:

Proposition 6 *Let $p_g > 0$. Then: a) The interval $I = \left[\underline{S}_{p_g}, \overline{S}\right] \neq \emptyset$; b) for $S \in I$ both DAE and IEA types of equilibria exist; c) if $p_g > \underline{p_g}$ there exists $\underline{\delta}_{p_g p_u}$ such that for $\delta \in \left(\underline{\delta}_{p_g p_u}, 1\right)$ and $S > \overline{S}$ DAE outcomes yield g and m joint payoffs greater than p's payoff in the (unique SPE) bilateral bargaining game over $1 + S$.*

In short, then, although high stakes generate inefficiencies in the sense that agreement is delayed, delays may still be preferred by the public sector to centralised bargaining.

The main conclusions of this section may be summarised as follows:

- Provided the government stakes are not too high, there exist equilibria with immediate agreement in which the union manages to appropriate part of the resources available;
- if government stakes are not too low, delayed agreement can be supported in equilibria;
- the aggregate ex ante expected surplus is higher for the public sector (government plus management) in IAE as opposed to DAE; however, IAE may fail to exist. If this is the case, though:
- the public sector may be better off under DAE as opposed to bilateral bargaining over $(S + 1)$. This is true when the union is sufficiently patient, in which case under bilateral negotiations it would manage to appropriate most of the surplus.

4 Uncertainty

The results derived in the previous section apply to a situation in which all parties are perfectly informed about the surplus available in case of agreement as well as the stake. However, in many situations it is more appropriate to assume that some form of uncertainty is present. In this section we model the situation where there is uncertainty over both the surplus available in the bilateral bargaining and the value of the stake. The stake is $S-s$, where s is a random variable uniformly distributed in $[0,A]$, $A>S$ whose realization is privately known by the stakeholder. The bilateral surplus is $1-b$, where b is a random variable uniformly distributed in $[0,B]$, $B>1$. Only the bargainers observe the realization of b.

Bargainers can reach a bilateral agreement at any t, $t=k\Delta, k=0,2,4,...$ obtaining a payoff $\frac{1-b}{2}e^{-k\Delta}$ each, and giving the stakeholder a payoff $(S-s)\,e^{-k\Delta}$; or they can delay in the hope that the stakeholder enters and a multilateral negotiation takes place. The stakeholder can wait for a bilateral agreement, or she may enter multilateral negotiation at any $t=k\Delta, k=1,3,5,....$ Upon entry at t, all private information is revealed and the bargaining game over $S+1-b-s$ yields an immediate agreement in which each obtains $\frac{S+1-b-s}{3}e^{-t}$.

For simplicity we treat the bargainers as a single player. A *strategy of the stakeholder* is a function t^s selecting, for each reservation value $s\in[0,A]$, a date $t^s(s)$ to enter into a multilateral negotiation. A *strategy of the bargainers* is a function t^b selecting, for each reservation value $b\in[0,B]$, a bilateral agreement date $t^b(b)$. We say that *player i of type x yields at t* iff $t^i(x)=t$. Given a pair of strategies $\left(t^s,t^b\right)$, let $P^s(t)$ and $P^b(t)$ denote the probabilities that a multilateral negotiation takes place and respectively, that a bilateral agreement is attained, at a date $\tau\leq t$. A *Bayesian Equilibrium* (BE) is a pair of strategies (t^s,t^b) such that for all $b\in[0,B]$ and all $s\in[0,A]$,

$$t^b(b)=\arg\max\int_0^t\frac{S+1-s(\tau)-b}{3}e^{-\tau}dP^s(\tau)+(1-P^s(t))\,\frac{(1-b)}{2}e^{-t}, \tag{4}$$

$$t^s(s)=\arg\max\int_0^t(S-s)\,e^{-\tau}dP^b(\tau)+\Big(1-P^b(t)\Big)\,\frac{S+1-s-b_t\,(s)}{3}e^{-t}, \tag{5a}$$

where $b_t\,(s)=E(b|b(t)<b\leq S+1-s)$.

We consider the game in the limit as $\Delta\rightarrow0$, so that $t^b(b)$ and $t^s(b)$ take values in the interval $[0,\infty]$, and look for BE in strategies that are continuously differentiable [22] in t.

[22] This is a mild assumption since it is not hard to show that Lipschitz continuity of the strategies (and thus

A *strategy is type-monotone* iff $t^i(x) \leq t^i(x')$ for all $x < x'$, where the inequality is strict unless $t^i(x) = 0$. Our first observation is that BE strategies must be type-monotone.

Lemma 7 *BE strategies are type monotone.*

Proof. See Appendix.

Observe that when the stakeholder yields the resulting payoffs depend on the type of both players; and consequently depend on the strategy profile. Since BE profiles are monotone in type, over time the relative value of a concession increases for bargainers and decreases for the stakeholder. This feature distinguishes the present game from more standard models of the war of attrition in which the payoffs are stationary.

Given type-monotonicity, a BE strategy profile $\left(t^s, t^b\right)$ is fully characterized by continuously differentiable and strictly increasing functions $(s(.), b(.))$,the inverse functions $\left(t^s, t^b\right)$ mapping dates into types such that $(s(t), b(t)) = (x, y)$ iff $t^s(x) = t$ and $t^b(y) = t$. Moreover, $P^s(t) = \frac{s(t)}{A}$ and $P^b(t) = \frac{b(t)}{B}$.

Before we characterize equilibrium $(s(t), b(t))$ at $0 < t < \infty$, $t^i(x)$,the following two observations are obvious but important.

Lemma 8 *At any BE strategy profile $t^s(s) = \infty$ for all $s > S$ and $t^b(b) = \infty$ for all $b > 1$.*

Lemma 9 *At any a BE strategy profile, $P_s(0) P_b(0) = 0$.*

Lemma 8 simply states that types that can not generate any surplus have a dominant strategy, namely never to yield. Lemma 9 points out that if there is a strictly positive probability that the opponent will yield at the start of the game, then any type planning to yields at $t = 0$ must benefit from a deviation in which she waits to see if the opponent does yield.

We are now ready to state our main result for this section.

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differentiability almost everywhere) is a necessary condition for BE. See [23] for a proof that differentiability is necessary for BE in a very related model.

Proposition 10 *The following strategy profile $\left(t^{s*}, t^{b*}\right)$ constitutes the unique BE:*

$$t^{s*}(s) = \begin{cases} 0 \text{ iff } s < s(0), \\ t \text{ iff } s = s(t), \\ \infty \text{ iff } s \geq S; \end{cases}$$

$$t^{b*}(s) = \begin{cases} 0 \text{ iff } b < b(0), \\ t \text{ iff } b = b(t), \\ \infty \text{ iff } b \geq 1; \end{cases}$$

where $(s(.), b(.))$, that are strictly increasing, are the unique solution to

$$\begin{aligned} s' &= \tfrac{3(A-s)(1-b)}{2(S-s)-(1-b)}, \\ b' &= \tfrac{(B-b)(1+S-s-b)}{5(S-s)-(1+B-2b)}, \end{aligned} \tag{5b}$$

such that

$$b(0) \times s(0) = 0, \tag{5c}$$

and

$$\lim_{t\to\infty} b(t) = 1,\ \lim_{t\to\infty} s(t) = a \equiv S - \frac{B-1}{5}. \tag{5d}$$

Proof. Fix a BE. By Lemmas 8 and 9 there must be types such that $t^i(x) \in (0, \infty)$ and a first order condition is necessary for these types. Differentiating 4 with respect to t we obtain first order condition for the bargainer of type $b\,(t)$:

$$\left[\frac{S+1-s(t)-b(t))}{3} - \frac{(1-b(t))}{2}\right]\frac{ds(t)}{dt} - (A-s(t))\,\frac{(1-b(t))}{2} = 0,$$

$$\left[\frac{2(S-s(t))-(1-b(t))}{3}\right]\frac{ds(t)}{dt} = (A-s(t))\,(1-b(t)),$$

$$\frac{ds(t)}{dt} = \frac{3\,(A-s(t))\,(1-b(t))}{[2(S-s(t))-(1-b(t))]}. \tag{8a}$$

Similarly, differentiating 5a with respect to t, taking into account that $b_t\,(s) = \frac{1+S-s+b(t)}{2}$, we obtain the first order necessary condition for type $s(t)$:

$$\begin{aligned} &\left[(S-s\,(t)) - \frac{(S-s\,(t))+(1-b\,(t))}{6}\right]\frac{db(t)}{dt} \\ =\ &\frac{B-b(t)}{6}\left[(S-s\,(t))+(1-b\,(t))+\frac{db(t)}{dt}\right] \end{aligned}$$

$$\left[(S - s(t)) - \frac{(S - s(t)) + (1 - b(t))}{6} - \frac{B - b(t)}{6}\right] \frac{db(t)}{dt}$$
$$= \frac{B - b(t)}{6} [(S - s(t)) + (1 - b(t))]$$

$$\left[\frac{5(S - s(t)) - (1 + B - 2b(t))}{6}\right] \frac{db(t)}{dt}$$
$$= \frac{B - b(t)}{6} [(S - s(t)) + (1 - b(t))]$$

$$\frac{db(t)}{dt} = \frac{(B - b(t)) [(S - s(t)) + (1 - b(t))]}{5(S - s(t)) - (1 + B - 2b(t))} \tag{8b}$$

Hence a BE must be characterized by a solution to the autonomous dynamical system 5b.

Moreover, a relevant solution of 5b must be strictly increasing, by Lemma 7, and by Lemma 9 must have initial condition in the set $I = \{(x, y) \in \Re^2 \text{ such that } xy = 0\}$. Consider the open set D,

$$D = \{(x, y) \in (-\epsilon, 1) \times (-\epsilon, S),$$
$$\text{such that } y < \min\left\{S - \tfrac{1}{2} + \tfrac{x}{2}, S - \tfrac{1+B}{5} + \tfrac{2x}{5}\right\}\}$$

Note that any solution of 5b such that $(s(t), b(t)) \notin D$ for some t, $0 < t < \infty$, cannot characterize a BE strategy profile, either because it is decreasing, in contradiction with Lemma 7 or because it prescribes that types $s > S$ or $b > 1$ yield, contradicting Lemma 8. On the other hand, by the Fundamental Theorem of ordinary differential equations[23], a unique solution to 5b goes though each $(x, y) \in I \cap D$. And observe moreover that each such solution is strictly increasing, and approaches the boundary of D. Consider the point $(1, a)$, $a = S - \frac{B-1}{5}$ in the boundary of D, and observe that there is a unique $(x, y) \in I \cap D$ such that the solution to 5b through (x, y) approaches $(1, a)$. Denote this unique point (σ^*, β^*) and let (b^*, g^*) denote the unique solution to 5b with initial condition (σ^*, β^*). Observe that and for all t, $0 < t < \infty$, $b^*(t) < 1$ and $s^*(t) < a$, and $\lim_{t\to\infty} b^*(t) = 1, \lim_{t\to\infty} s^*(t) = a$.

We claim that if there is a BE, it must be characterized by (b^*, s^*).

Consider any other solution to 5b $\left(\widetilde{s}, \widetilde{b}\right)$, with initial condition $(\sigma, \beta) \in I \cap D$, $(\sigma, \beta) \neq (\sigma^*, \beta^*)$, and let us check that it cannot characterize a BE. Since $\left(\widetilde{s}(t), \widetilde{b}(t)\right)$ approaches the boundary of B at $(z, u) \neq (1, a)$, then either i) $z = 1, u < S$; or ii) $z < 1$, $u = S - \frac{1+B}{5} + \frac{2z}{5}$; or iii) $z < 1, u = S - \frac{1}{2} + \frac{z}{2}$. In case i) $\left(\widetilde{s}(T), \widetilde{b}(T)\right) = (z, u)$ for $T < \infty$,that is, T is the last date at which any type of either player yields. But this is incompatible with BE behaviour since any

[23] See Hirsch and Smale [1974] ,page 162.

type s , $s(T) < s < S$, such that $\widetilde{t}^s(s) = \infty$ according to the alleged strategy, is strictly better off deviating to $t'^s(s) = T + \Delta$. In cases ii) and iii) $\widetilde{t}^b(b) = \infty$ for $b < 1$. Along this profile, for each $\pi > 0$,there is a $t_\pi < \infty$, such that $P\left(\widetilde{t}^s(s) < \infty | \widetilde{t}^s(s) \geq t_\pi\right) \leq \pi$; and for each $b < 1$ there is a $\pi_b > 0$ such that if $P\left(\widetilde{t}^s(s) \in [t, \infty) | \widetilde{t}^s(s) \geq t\right) \leq \pi_b$, then $\widetilde{t}^b(b) = \infty$ cannot maximize b's expected gains because

$$\int_t^\infty \frac{S + 1 - s(\tau) - b}{3} e^{-\tau} dP^s(\tau) \leq \pi_b \frac{S + 1 - s(\tau) - b}{3} e^{-t} < \frac{(1 - b)}{2} e^{-t},$$

contradicting that $\widetilde{t}^b(b) = \infty$ is a best response for any $b < 1$.

To complete the proof we check that at $\left(t^{s*}, t^{b*}\right)$ the necessary conditions are indeed sufficient for a BE, i.e. that each type indeed maximizes her expected payoff given that the opponent plays the alleged strategy. By Lemma 31a (see Appendix) the first order conditions are indeed sufficient for a maximum for all types such that $t^{*b}(b) < \infty$ and $t^{*s}(s) < \infty$. Observe moreover, that since for each $t < \infty$ there are types $s < a$ maximizing her payoff by $t^{*s}(s) = t$, by monotonicity any $s \geq a$ must maximize her payoff with $t^{*s}(s) \geq \sup\{t^{*s}(s), s < a\} = \infty$. Therefore $\left(t^{s*}, t^{b*}\right)$ is indeed a BE. ■

The following are now immediate:

Corollary 11 *If the maximum stake S is not too large, bargainers reach a bilateral agreement at $t = 0$ with positive probability, and if the stakeholder enters a multilateral negotiation it does so with delay. When the uncertainty about the bilateral bargaining is small (large), $B \leq \frac{3}{2}$ ($B > \frac{3}{2}$, resp.), it is sufficient that $S \leq \frac{1}{2}$ ($S \leq \frac{1+B}{5}$, resp.).*

The above confirms the outcome of Proposition 3, in that immediate agreement may be possible even when there is uncertainty, and actually in this case precisely because of that, and is supported by the fact that the stakeholder would let the bargainers to "fight it out" before intervening. Indeed, the larger the (maximum) surplus bargained over, the less likely it is that the stakeholder intervenes:

Corollary 12 *The probability that the stakeholder with positive stake enters a multilateral negotiation decreases with B, and is never greater than $\frac{a}{S} = \frac{1}{5}\frac{5S+1-B}{S}$.*

5 Discussion

In our analysis we have found that as long as the stakes are sufficiently high inefficiencies abound, in the sense that delayed agreements obtain in equilibrium with positive probability, and overall resources are wasted [24]. This result may initially lead to conclude that the stakeholder would be better off by not getting directly entangled into negotiations. However, direct involvement allows the government to collude with the management and as long as the union is sufficiently weak (which in our model corresponds to decreasing the probability p_u that the union makes an offer), the ex-ante payoff to the public sector in a delayed agreement equilibrium is higher than if the stakeholder 'centralised' negotiations. That is, a weak union may make it worthwhile for the government to intervene in public sector negotiations: the government can even provide positive contributions[25] and still be better off as compared to a situation of straightforward bilateral bargaining between the public sector employer and the union (over an aggregate surplus of $S + 1$).

Furthermore, if confronted with a weak union, the government can hold out in negotiations, which is consistent with our war of attrition model, and is confirmed by actual events. For instance, in the UK the national strike in the fire services in 1977-78 lasted for nine weeks and ended mostly as it became clear that the government, not directly involved in the negotiations, would not intervene[26].

Our model with uncertainty also provides a rationale for the introduction of advisory bodies which make salary recommendations to the government as an employer[27]. In situations were central government is not a *direct* employer, in practice the effect of such bodies is to remove a potentially sympathetic[28] management (the *direct* employer) from salary negotiations altogether. Moreover, because such bodies have a purely advisory role, as a result the government has total freedom in its decision whether or not to accept the recommendation. In terms of our framework,

[24] A case in point is Canada, where strike activity in the public sector is higher than in the private sector. Strike days lost typpically account for 20-30% of all strike activity, and in1991 they reached a peak of 57%. See [12].

[25] Although we do not model it, this also brings additional political benefits due to a change in voter's perception of the dispute.

[26] The strike was to bring salaries in the fire service in line with those of other energy workers (i.e. gas and water). Although no salary agreement was reached as a result of the strike, the fire service did obtain an undertaking to bring pay up to the upper quartile of manual workers earnings by November 1979 and to mantain it at that level thereafter. See [19].

[27] For instance, in the UK Pay Review Bodies are committees of experts appointed by the Prime Minister which make mainly salary recommendations for various categories of workers in the public sector (see e.g [10] and [9]). In Italy a Commission of nine experts is appointed by the President of the Republic, following indications from the two legislative Chambers (see [26]).

[28] A case in point is the UK Fire Service industrial dispute in 1980. A pay increase of 18.8% was reached when the Labour party gained control of the employer's side of the National Joint Council under a Conservative government (see [19]).

bargainers' failure to reach an immediate agreement is equivalent to a call for a pay review; the government's acceptance of a recommendation (which "fixes" the random variable s) corresponds to our stakeholder "entering" negotiations[29].

Our analysis can therefore provide a rationale for industrial relation legislation across European nations; however, our conclusions can be extended to more general situations where bilateral conflict has the potential to involve wider interests. Our results seem to suggest that generally the presence of a stakeholder introduces inefficiencies; yet, wherever there is the potential for a coalition of the stakeholder and the one of the bargainers to form, these inefficiencies may be exploited strategically and benefit the members of the coalition. Finally, in the presence of uncertainty the fact that the stakeholder may delay his involvement supports equilibria were bargainers agree immediately.

6 Appendix

6.1 Proof of Proposition 1

We first show that there is a unique stationary equilibrium with immediate agreement, then exhibit the profile of supporting strategies. As a preliminary, let $x^i = \left(x_1^i, x_2^i\right)$ and $c^i = \left(c_1^i, c_2^i\right)$ denote the surplus division and the contribution vector, respectively, proposed by agent i when he is selected to make a proposal, where $x_1^i + x_2^i = 1$ and $c_1^i + c_2^i \leq S$ $(i = 1, 2, s)$. Furthermore, let $\gamma^i = c_1^i + c_2^i$. It is useful to start by defining each agent ex-ante equilibrium payoff. Let $i = 1, 2$; then:

$$\begin{array}{c} V_i = \frac{1}{3}\left(x_i^i + c_i^i\right) + \frac{1}{3}\left(x_i^j + c_i^j\right) + \frac{1}{3}\left(x_i^s + c_i^s\right) \\ V_s = \frac{1}{3}\left(S - \gamma^s\right) + \sum_{i=1,2} \frac{1}{3}\left(S - \gamma^i\right) \end{array} \tag{9}$$

at an equilibrium optimality and stationarity require:

$$\begin{array}{c} x_i^j + c_i^j = \delta_i V_i,\ i, j = 1, 2 \\ S - \gamma^i = \delta_s V_s \end{array} \tag{10}$$

The above summarises a system of six equations. The first line defines the optimal proposal in both subgames where bargainer i is a responder (either to the other bargainer or the stakeholder),

[29] In actual fact British Governments have hardly rejected any recommendation from a Pay Review Body. However, it is pretty common to either implement only a subset of the recommendations (which may cover a number of other issues additional to salaries) or just delay taking any decision. This delaying tactics is in effect tantamount to rejecting the reccommendations, although it carries less political stigma than a straight rejection - a feature which our model allows for. See [5].

whereas the second line defines the optimal contributions in both subgames where the stakeholder is a responder (to either one of the bargainers).

System 10 in the text implies

$$x_i^j + c_i^j = x_i^s + c_i^s \tag{11a}$$

and

$$S - \gamma^1 = S - \gamma^2 \tag{11b}$$

So let $\gamma^1 = \gamma^2 = \gamma$. Substituting the expressions for the ex ante expected payoffs (in 9) yields

$$\begin{array}{c} x_i^j + c_i^j = \frac{1}{3}\delta_i \left(x_i^i + c_i^i\right) + \frac{1}{3}\delta_i \left(x_i^j + c_i^j\right) + \frac{1}{3}\delta_i \left(x_i^s + c_i^s\right) \\ S - \gamma = \delta_s \left(\frac{1}{3}\left(S - \gamma^s\right) + \sum_{i=1,2} \frac{1}{3}\left(S - \gamma\right)\right) \end{array}$$

which because of equations 11a and 11b can be rewritten as

$$\begin{array}{c} \left(1 - \frac{2}{3}\delta_i\right)\left(x_i^j + c_i^j\right) = \frac{1}{3}\delta_i \left(x_i^i + c_i^i\right) \\ \left(S - \gamma\right)\left(1 - \frac{2}{3}\delta_s\right) = \frac{1}{3}\delta_s \left(S - \gamma^s\right) \end{array}$$

from which

$$x_i^i = \frac{\left(3 - 2\delta_i\right)\left(x_i^j + c_i^j\right)}{\delta_i} - c_i^i \tag{11c}$$

$$\gamma^s = \frac{\left(3 - 2\delta_s\right)\gamma - 3\left(1 - \delta_s\right)S}{\delta_s} \tag{11d}$$

Recalling the efficiency requirement $x_1^i + x_2^i = 1$, and noting that $x_i^j + c_i^j = 1 - x_j^j + \left(\gamma - c_j^j\right)$, the above equations yield

$$\begin{array}{c} x_1^1 = \frac{(3-2\delta_1)\left(1-x_2^2+\left(\gamma-c_2^2\right)\right)}{\delta_1} - c_1^1 \\ x_2^2 = \frac{(3-2\delta_2)\left(1-x_1^1+\left(\gamma-c_1^1\right)\right)}{\delta_2} - c_2^2 \\ \gamma^s = \frac{(3-2\delta_s)\gamma-3(1-\delta_s)S}{\delta_s} \end{array}$$

The first two equations can be solved to yield

$$\begin{array}{c} x_1^1 = \frac{(1-\delta_2)(3-2\delta_1)(1+\gamma)}{3-2\delta_1-2\delta_2+\delta_1\delta_2} - c_1^1 \\ x_2^2 = \frac{(1-\delta_1)(3-2\delta_2)(1+\gamma)}{3-2\delta_1-2\delta_2+\delta_1\delta_2} - c_2^2 \end{array} \tag{11e}$$

From equations 11a recall that $x_i^s = x_i^j + c_i^j - c_i^s = 1 - x_j^j + \left(\gamma - c_j^j\right) - c_i^s$. Then, because of the efficiency requirement $x_i^s + x_j^s = 1$ we must have

$$\begin{array}{c} 1 - x_2^2 + \left(\gamma - c_2^2\right) - c_1^s + 1 - x_1^1 + \left(\gamma - c_1^1\right) - c_2^s = 1 \Leftrightarrow \\ 2\left(1 + \gamma\right) - x_2^2 - c_2^2 - x_1^1 - c_1^1 - \gamma^s = 1 \end{array}$$

Substituting the values for 11e in the last expression and coupling this with 11d we can solve the resulting two equations

$$\gamma^s = \frac{(3-2\delta_s)\gamma - 3(1-\delta_s)S}{\delta_s}$$
$$2\left(1+\gamma\right) - \left(\frac{(1-\delta_1)(3-2\delta_2)(1+\gamma)}{3-2\delta_1-2\delta_2+\delta_1\delta_2} - c_2^2\right) - c_2^2 - \left(\frac{(1-\delta_2)(3-2\delta_1)(1+\gamma)}{3-2\delta_1-2\delta_2+\delta_1\delta_2} - c_1^1\right) - c_1^1 - \gamma^s = 1$$

to obtain the expressions for $\widehat{\gamma}$ and $\widehat{\gamma}^s$, that is:

$$\boxed{\begin{aligned} \widehat{\gamma}^s &= \frac{((1-\delta_s)(\delta_1+\delta_2-2\delta_1\delta_2))S-(3-2\delta_s)(1-\delta_1)(1-\delta_2)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} \\ \widehat{\gamma} &= \frac{(1-\delta_s)(3-2\delta_2-2\delta_1+\delta_1\delta_2)S-\delta_s(1-\delta_1-\delta_2+\delta_1\delta_2)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} \end{aligned}}$$

which can now be substituted back into 11e to yield

$$\boxed{\begin{aligned} x_1^1 &= \frac{(1-\delta_2)(1-\delta_s)(3-2\delta_1)(1+S)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} - c_1^1 = \widehat{x}_1^1 \\ x_2^2 &= \frac{(1-\delta_1)(1-\delta_s)(3-2\delta_2)(1+S)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} - c_2^2 = \widehat{x}_2^2 \end{aligned}}$$

from which the responder bargaining share can be derived:

$$1-\widehat{x}_j^j = \frac{(1-\delta_j)(\delta_s+\delta_i-2\delta_i\delta_s)-(3-2\delta_j)(1-\delta_i)(1-\delta_s)S}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} + c_j^j$$

Note that $\widehat{\gamma^s} - \widehat{\gamma} < 0$, so that the contribution that the stakeholder concedes when he is a proposer is less than the contribution he concedes when responding to a bargainer, since

$$\widehat{\gamma^s} - \widehat{\gamma} = \frac{-3\left(1-\delta_s\right)\left(1-\delta_1\right)\left(1-\delta_2\right) - 3\left(1-\delta_1-\delta_2+\delta_1\delta_2\right)S - \delta_s\left(\delta_1+\delta_2-2\delta_1\delta_2\right)S}{3-\delta_s\left(2-\delta_2\right)-\delta_1\left(2-\delta_s\right)-\delta_2\left(2-\delta_1\right)} < 0$$

Furthermore, $\widehat{\gamma^s} - \widehat{\gamma} < S$, so that the bargainers do not manage to extract the whole surplus from the stakeholder. Note also that we need $\widehat{\gamma^s} - \widehat{\gamma} \geq 0$. Since the denominator in the above expression is positive[30], this requirement reduces to

$$S \geq \frac{\delta_s(1-\delta_1-\delta_2+\delta_1\delta_2)}{(1-\delta_s)(3-2\delta_2-2\delta_1+\delta_1\delta_2)} = \underline{S}$$
$$S \geq \frac{(3-2\delta_s)(1-\delta_1)(1-\delta_2)}{(1-\delta_s)(\delta_1+\delta_2-2\delta_1\delta_2)} = \underline{S}_s$$

where $\underline{S}_s \geq \underline{S}$, where this last inequality holds true since

$$\underline{S}_s = \frac{\left(3-2\delta_S\right)\left(1-\delta_1\right)\left(1-\delta_2\right)}{\left(1-\delta_S\right)\left(\delta_1+\delta_2-2\delta_1\delta_2\right)} \geq \frac{\delta_S\left(1-\delta_1-\delta_2+\delta_1\delta_2\right)}{\left(1-\delta_S\right)\left(3-2\delta_2-2\delta_1+\delta_1\delta_2\right)}\underline{S}$$

[30] The denominator is monotonically decreasing in the discount factors: let $D\left(\delta_1,\delta_2,\delta_S\right) 3-\delta_s\left(2-\delta_2\right)-\delta_1\left(2-\delta_s\right)-\delta_2\left(2-\delta_1\right)$. Then $\frac{\partial(D(\delta_1,\delta_2,\delta_S))}{\partial\delta_i} = \delta_j - 2 + \delta_k < 0$, where $i,j,k = 1,2,S$. So, the smallest value is for $\delta_i \to 1$ for all i, which the denominator tends to 0. Thus we only need to ensure that the numerator in the expressions for $\widehat{\gamma}$ and $\widehat{\gamma^S}$ is positive, which yields the expressions in the text.

if and only if

$$\left(3-2\delta_S\right)\left(1-\delta_1\right)\left(1-\delta_2\right)\left(3-2\delta_2-2\delta_1+\delta_1\delta_2\right) \geq \delta_S\left(1-\delta_1-\delta_2+\delta_1\delta_2\right)\left(\delta_1+\delta_2-2\delta_1\delta_2\right)$$

that is if and only if

$$3\left(1-\delta_2\right)\left(1-\delta_1\right)\left(3+\delta_1\left(\delta_2-2\right)+\delta_S\delta_1-2\delta_2+\delta_S\left(\delta_2-2\right)\right) \geq 0$$

where the last inequality is always satisfied.

Above we have derived the equilibrium shares when the two bargainers are proposers. The corresponding equilibrium *payoffs* (share plus contribution) are

$$\begin{aligned}
\widehat{x}_i^i + c_i^i &= \frac{\left(1-\delta_j\right)\left(1-\delta_s\right)\left(3-2\delta_i\right)}{3-\delta_s\left(2-\delta_2\right)-\delta_1\left(2-\delta_s\right)-\delta_2\left(2-\delta_1\right)}\left(1+S\right) = \pi_{Pi} \\
1-\widehat{x}_j^j + \left(\widehat{\gamma}-c_j^j\right) &= \delta_i\frac{\left(1-\delta_j\right)\left(1-\delta_s\right)}{3-\delta_s\left(2-\delta_2\right)-\delta_1\left(2-\delta_s\right)-\delta_2\left(2-\delta_1\right)}\left(1+S\right) = \widehat{x}_i^s + c_i^s = \pi_{Ri}
\end{aligned}$$

Note that the equilibrium payoff of a proposing bargainer is the same for both, as is the responding bargainer payoff. Correspondingly, ex ante equilibrium payoffs are while ex ante equilibrium payoffs are

$$\begin{aligned}
\widehat{V}_i &= \tfrac{(1-\delta_j)(1-\delta_s)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)}\left(1+S\right) \\
\widehat{V}_s &= \tfrac{(1-\delta_1)(1-\delta_2)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)}\left(1+S\right)
\end{aligned}$$

Consequently it must be that the bargaining agreement proposed by the stakeholder is such that

$$\widehat{x}_i^s + c_i^s = \delta_i\widehat{V}_i$$

Finally, we have to ensure that equilibrium shares are feasible, that is $\widehat{x}_i^i \in [0,1]$. This is true as long as

$$\underline{b_i} = \tfrac{(1-\delta_s)(3-2\delta_i)(1-\delta_j)S-(1-\delta_i)(\delta_s+\delta_j-2\delta_s\delta_j)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} \leq c_i^i \leq \tfrac{(1-\delta_s)(3-2\delta_i)(1-\delta_j)(S+1)}{3-\delta_s(2-\delta_2)-\delta_1(2-\delta_s)-\delta_2(2-\delta_1)} = \overline{b_i}$$

Together with the requirement $c_i^s + c_i^s = \widehat{\gamma}^s$, the above establishes a family of equilibrium offers. It is then straightforward to verify that the strategies in the statement of Proposition 1 are indeed an equilibrium. ■

Note that to compute the stationary equilibrium shares in trilateral bargaining over $1+S$ the only change is that now $x_i^j + c_i^j = 1 + S - x_j^j - c_j^j$ (rather than $x_i^j + c_i^j = 1 - x_j^j + \left(\gamma - c_j^j\right)$), so that we now have

$$\begin{aligned} x_1^1 &= \tfrac{(3-2\delta_1)\left(1+S-x_2^2-c_2^2\right)}{\delta_1} - c_1^1 \\ x_2^2 &= \tfrac{(3-2\delta_2)\left(1+S-x_1^1-c_1^1\right)}{\delta_2} - c_2^2 \\ \gamma^s &= \tfrac{(3-2\delta_s)\gamma-3(1-\delta_s)S}{\delta_s} \end{aligned}$$

where the first two equations can be solved to yield:

$$x_i^i = \frac{(3-2\delta_i)(1-\delta_j)(1+S)}{3-2\delta_1-2\delta_2+\delta_2\delta_1} - c_i^i > \widehat{x}_i^i.$$

6.2 Proof of Proposition 2.

As before, let V_i denote the expected payoff to bargainer i. We will show that it is not possible to construct an equilibrium in which an agent's proposal is (always) rejected.

Consider first an equilibrium in which the stakeholder's proposal is always rejected. Then, considering that with probability $\frac{1}{3}$ the stakeholder is chosen as a proposer - and his proposal rejected - V_i satisfies

$$V_i = \tfrac{1}{3}\left(x_i^i + c_i^i\right) + \tfrac{1}{3}\left(1 - x_j^j + \gamma^j - c_j^j\right) + \tfrac{1}{3}\delta_i V_i \Leftrightarrow V_i = \tfrac{x_i^i + c_i^i + \left(1-x_j^j+\gamma^j-c_j^j\right)}{3-\delta_i}$$

where $\left(x_i^i + c_i^i\right)$ and $\left(1 - x_j^j + \gamma^j - c_j^j\right)$ are bargainer i's payoffs when proposing and when responding, respectively. For the stakeholder we have

$$V_s = \tfrac{1}{3}\delta_s V_s + \tfrac{1}{3}\left(S-\gamma^i\right) + \tfrac{1}{3}\left(S-\gamma^j\right) \Leftrightarrow V_s = \tfrac{\left(S-\gamma^i\right)+\left(S-\gamma^j\right)}{3-\delta_s}$$

In equilibrium the offers by the two bargainers will be accepted by their opponent only if

$$\begin{aligned} 1 - x_j^j + \left(\gamma^j - c_j^j\right) &= \delta_i V_i = \delta_i \tfrac{x_i^i+c_i^i+\left(1-x_j^j+\gamma^j-c_j^j\right)}{3-\delta_i} \\ \left(S-\gamma^i\right) = \left(S-\gamma^j\right) &= \delta_s V_s = \delta_s \tfrac{\left(S-\gamma^i\right)+\left(S-\gamma^j\right)}{3-\delta_s} \end{aligned}$$

The last equation implies $\gamma^1 = \gamma^2 = \gamma = S$, so that the bargainers manage to appropriate the whole surplus. Substitution of into $\gamma = S$ into the first two equation yields

$$\begin{aligned} x_1^1 &= \tfrac{(1-\delta_2)(3-2\delta_1)(1+S)}{3-2\delta_1-2\delta_2+\delta_1\delta_2} - c_1^1 \\ x_2^2 &= \tfrac{(1-\delta_1)(3-2\delta_2)(1+S)}{3-2\delta_1-2\delta_2+\delta_1\delta_2} - c_2^2 \end{aligned}$$

with corresponding equilibrium payoffs

$$\begin{aligned} x_i^i + c_i^i &= \tfrac{(1-\delta_j)(3-2\delta_i)}{3-2\delta_1-2\delta_2+\delta_1\delta_2}(1+S) \\ 1 - x_j^j + S - c_j^j &= \tfrac{\delta_j(1-\delta_i)}{3-2\delta_1-2\delta_2+\delta_1\delta_2}(1+S) \end{aligned}$$

whereas ex ante expected payoffs are

$$V_i = \frac{(1+S)}{3-\delta_i}$$
$$V_s = 0$$

For the above equilibrium to be supported the stakeholder must have no incentive to deviate and make an offer that would be accepted. So we require

$$(1+S) - \delta_1 \frac{(1+S)}{3-\delta_1} - \delta_2 \frac{(1+S)}{3-\delta_2} \leq 0 = \delta_s V_s \Leftrightarrow 3\frac{3-2\delta_2-2\delta_1+\delta_1\delta_2}{(3-\delta_1)(3-\delta_2)} \leq 0$$

which is never true.

Let us now try to construct an equilibrium in which bargainer 1's proposal is always rejected. Then ex ante equilibrium payoffs must satisfy

$$V_1 = \tfrac{1}{3}\delta_1 V_1 + \tfrac{1}{3}\left(1 - x_2^2 + \gamma^2 - c_2^2\right) + \tfrac{1}{3}\left(x_1^s + c_1^s\right) \Leftrightarrow V_1 = \frac{1-x_2^2+\gamma^2-c_2^2+x_1^s+c_1^s}{3-\delta_1}$$
$$V_2 = \tfrac{1}{3}\delta_2 V_2 + \tfrac{1}{3}\left(x_2^2 + c_2^2\right) + \tfrac{1}{3}\left(x_2^s + c_2^s\right) \Leftrightarrow V_2 = \frac{x_2^2+c_2^2+x_2^s+c_2^s}{3-\delta_2}$$

For the stakeholder we have

$$V_s = \tfrac{1}{3}\delta_s V_s + \tfrac{1}{3}\left(S - \gamma^2\right) + \tfrac{1}{3}\left(S - \gamma^s\right) \Leftrightarrow V_s = \frac{\left(S-\gamma^2\right)+\left(S-\gamma^s\right)}{3-\delta_s}$$

In equilibrium for bargainer 2's offer to be accepted it must be that

$$1 - x_2^2 + \left(\gamma^2 - c_2^2\right) = \delta_1 V_1 = \delta_1 \frac{1-x_2^2+\gamma^2-c_2^2+x_1^s+c_1^s}{3-\delta_1}$$
$$\left(S - \gamma^2\right) = \delta_s V_s = \delta_s \frac{\left(S-\gamma^2\right)+\left(S-\gamma^s\right)}{3-\delta_s}$$
$$1 - x_1^s + \gamma^s - c_2^s = \delta_2 V_2 = \delta_2 \frac{x_2^2+c_2^2+1-x_1^s+\gamma^s-c_2^s}{3-\delta_2}$$

so that

$$1 - x_2^2 + \gamma^2 - c_2^2 = \frac{\delta_1\left(x_1^s+c_1^s\right)}{(3-2\delta_1)}$$
$$\left(S - \gamma^2\right) = \delta_s \frac{\left(S-\gamma^s\right)}{3-2\delta_s}$$
$$1 - x_1^s + \gamma^s - c_1^s = \delta_2 \frac{x_2^2+c_2^2}{3-2\delta_2}$$

The second equation can be rearranged as

$$\gamma^2 = \frac{3\left(1-\delta_s\right)S + \delta_s\gamma^s}{3-2\delta_s}$$

which can be substituted into the other two expressions to solve for

$$x_2^2 + c_2^2 = \frac{(3-2\delta_2)}{3(3-2\delta_2-2\delta_1+\delta_1\delta_2)}\left(3\left(1-\delta_1\right) + \frac{3(\delta_s-\delta_1)}{(3-2\delta_s)}\gamma^s + \frac{3(3-2\delta_1)(1-\delta_s)}{(3-2\delta_s)}S\right)$$
$$x_1^s + c_1^s = \frac{(3-2\delta_1)}{3(3-2\delta_2-2\delta_1+\delta_1\delta_2)}\left(3\left(1-\delta_2\right) + \frac{3(3-2\delta_2-2\delta_s+\delta_2\delta_s)}{(3-2\delta_s)}\gamma^s - \frac{3\delta_2(1-\delta_s)S}{(3-2\delta_s)}\right)$$

Substituting these values into the discounted ex ante expected payoffs yields

$$\begin{aligned} \delta_1 V_1 &= \tfrac{\delta_1}{(3-2\delta_2-2\delta_1+\delta_1\delta_2)}\left((1-\delta_2)-\tfrac{\delta_2(1-\delta_s)}{(3-2\delta_s)}S+\tfrac{(3-2\delta_2-2\delta_s+\delta_2\delta_s)}{(3-2\delta_s)}\gamma^s\right) \\ \delta_2 V_2 &= \tfrac{\delta_2}{(3-2\delta_2-2\delta_1+\delta_1\delta_2)}\left((1-\delta_1)+\tfrac{(3-2\delta_1)(1-\delta_s)}{(3-2\delta_s)}S+\tfrac{(\delta_s-\delta_1)}{(3-2\delta_s)}\gamma^s\right) \\ \delta_s V_s &= \tfrac{\delta_s}{(3-2\delta_s)}\left(S-\gamma^s\right) \end{aligned}$$

Now for the above to be part of an equilibrium it must be that bargainer 1 has no incentive to deviate and profit from an offer which would be accepted, so that we require that

$$\begin{gathered} 1+S-\delta_2 V_2-\delta_s V_s<\delta_1 V_1 \Rightarrow \\ 3\left(1-\delta_2\right)\tfrac{(3-2\delta_s-3\delta_1+2\delta_s\delta_1)+(3-3\delta_s-2\delta_1+2\delta_s\delta_1)S+\gamma^s(\delta_s-\delta_1)}{(3-2\delta_2-2\delta_1+\delta_1\delta_2)(3-2\delta_s)}<0 \end{gathered}$$

which can never hold, even if $\delta_s<\delta_1$, given that $\gamma^s \leq S$. Thus, bargainer 1 would indeed have a profitable deviation. Consequently, no stationary equilibria with positive probability of delay exists. ■

6.3 Check of non-stationary equilibrium

To see that these strategies constitute an equilibrium, note that the corresponding ex-ante equilibrium expected payoffs at the time of agreement are

$$\begin{aligned} V_i &= \tfrac{1}{3}\left(\tfrac{2}{3}\right)+\tfrac{1}{3}\left(\tfrac{1}{3}\right)+\tfrac{1}{3}\left(\tfrac{1}{2}+\tfrac{1}{6}\right)=\tfrac{5}{9},\ i=1,2 \\ V_s &= \tfrac{1}{3}(S)+\tfrac{1}{3}(S)+\tfrac{1}{3}\left(S-\tfrac{1}{3}\right)=S-\tfrac{1}{9} \end{aligned}$$

We do not have to worry about the optimality of the responders' strategies, as deviations would be punished by forcing the deviator down in the following round to either 0 (if a bargainer) or[31] $\frac{7}{9}\delta_s S$ (if the stakeholder), so that they are not profitable. However, we have to ensure that no proposer has a profitable deviant offer which is in the interest of the responders to accept.

Any deviating proposal in the first round by bargainer i would be accepted by bargainer j if it yielded him a payoff in excess of $\delta_j V_j=\delta_j\frac{5}{9}$, leaving bargainer i with $1-\frac{5}{9}\delta_j<\delta_i V_i=\delta_i\frac{5}{9}$ if δ_i and δ_j are sufficiently high, that is if $\delta_i+\delta_j>\frac{9}{5}$. So, for sufficiently large values of the discount factors, a deviant proposal by bargainer i would be profitable only if i can obtain some positive contribution from the stakeholder, so that $1-\frac{5}{9}\delta_j+c_i>\delta_i\frac{5}{9}$. On the other hand, for

[31] It is easy to verify thatprovided that S is sufficiently large, the expected ex ante payoff in punishment is less than the equilibrium payoff to the stakeholder regardless of the proposer, that is: $\frac{7}{9}\delta_s S<\delta_s\left(S-\frac{1}{9}\right)$ for deviations in the first round (which requires $S>\frac{1}{2}$); and $\frac{7}{9}\delta_s S<S-\frac{1}{3}=\min\left\{S,S-\frac{1}{3}\right\}$ for deviations in the second round, at the time of agreement, which holds as long as $S\geq\frac{3}{2}$.

such a deviation to be successful, it must be profitable for the stakeholder to concede to such a demand, that is $S - c_i > \delta_s V_s = \delta_s \left(S - \frac{1}{9}\right)$. Putting these two inequalities together implies that

$$(\delta_i + \delta_j)\frac{5}{9} - 1 < c_i < S(1 - \delta_s) + \delta_s \frac{1}{9} \tag{21}$$

but as long as $S < \frac{1}{9}\frac{(5(\delta_i+\delta_j)-9-\delta_s)}{(1-\delta_s)} = \overline{S}$ this interval is empty[32].

Consider now deviations by the stakeholder in the first round. For a deviation to be profitable the contribution vector c offered to the bargainers (in order to bribe them into agreeing immediately) must be such that $S - c_1' - c_2' > \delta_s \left(S - \frac{1}{9}\right)$. Also, in order for the stakeholder's proposal to be accepted, it must be that for each agent $\frac{1}{2} + c_i' > \delta_i \frac{5}{9}$, or $1 + c_i' + c_j' > (\delta_i + \delta_j)\frac{5}{9}$. These inequalities can be rearranges as 21 , so that as long as $S < \overline{S}$ there are no profitable deviations for the stakeholder in the first round.

Consider now deviations at the time of agreement (second round). Recall that deviations are punished by reverting play in the following round to one in which the deviating agent receives either 0 (if a bargainer) or $\frac{7}{9}\delta_s S$ (if the stakeholder), and is always smaller than the equilibrium payoff, regardless of the identity of the proposer. Consequently, it is never profitable to deviate. Finally, observe that it is optimal to punish for the punishers, since failing to do so makes a failed punisher himself a deviator, which in turn calls for his own punishment, and correspondingly a smaller payoff than the equilibrium one. More precisely, consider first the case bargainer i has deviated, thereby triggering his punishment, and suppose that the same bargainer i is selected to make a proposal. Why is it optimal for him to propose $x_i = 0$ and $x_j = 1$? Surely he could try and bribe his opponents into agreement. However, he would have to offer the stakeholder at least $\frac{7}{9}\delta_s S$, thereby deviating to $c_j' = \frac{1}{9}S(9 - 7\delta_s)$; furthermore, he would have to ensure that the deviant offer x_j' is such that $x_j' + c_j' \geq \frac{1}{9}\delta_j(9 + 2S)$, that is he would have to offer at least $x_j' = \frac{1}{9}\delta_j(9 + 2S) - \frac{1}{9}S(9 - 7\delta_s) = \frac{1}{9}(9\delta_j - (9 - 2\delta_j - 7\delta_s)S) > 1$ if $\delta_s > \frac{9(1-\delta_j)+9S-2\delta_j S}{7S}$, which holds for sufficiently high values of S, so that no bribe is possible or profitable. Finally observe that it is optimal for the stakeholder to concede in the punishment phase as long as $\frac{2}{3}S > \frac{7}{9}\delta_s S \Leftrightarrow \delta_s < \frac{6}{7}$.

6.4 Proof of Lemma 7

The result is proved for the bargainers, a similar argument holds for the stakeholder.

Fix a BE profile. Let $t^b(b) = t$ and $t^b(b') = t'$, if $b > b'$, $t > 0$ and $t \neq t'$ then $t' < t$.

[32] Note that if δ_s is sufficiently large this is a very mild requirement.

Let

$$V(b,t) = \int_0^t \frac{S+1-Es(\tau)-b}{3} e^{-\tau} dP^s(\tau) + (1-P^s(t))\frac{(1-b)}{2}e^{-t}$$

and

$$\delta(t) = \left(\int_0^t e^{-\tau} dP^s(\tau) + (1-P^s(t))e^{-t} \right).$$

In a BE

$$V(b,t) \geq V(b,t') = V(b',t') + \left(b-b'\right)\delta(t'),$$

on the other hand

$$V(b',t') \geq V(b',t) = V(b,t) - \left(b-b'\right)\delta(t),$$

hence

$$V(b,t) \geq V(b,t) + \left(b-b'\right)\left(\delta(t') - \delta(t)\right),$$

and therefore

$$\left(\delta(t') - \delta(t)\right) \leq 0$$

which is equivalent to $t' < t$.

Lemma 13 *Given that s / b plays t^{s*}/ t^{b*}, for all $b^*(0) < b(t) < 1$ /$s^*(0) < s(t) < S$, the derivative of the expected gains is strictly decreasing at t.*

Observe that the first derivative of the payoffs of a bargainer of type b, can be written as $e^{-t}V(t)$,where

$$V(t) = \left(\frac{2(S-s(t))-(1-b)}{3} \right) \frac{ds}{dt} - (A - s(t))(1-b),$$

and thus the second derivative is of the form $Ve^{-t} + e^{-t}\frac{dV}{dt}$. Since $V(t) = 0$ at the singular point, it is sufficient to check that $\frac{dV}{dt}$is negative. Writing $V(t)$ as

$$V(t) = Z(t)s' - (A - s(t))(1-b)$$

the desired condition is

$$\frac{dW(t)}{dt} = s''Z + Z's' + s'(1-b) < 0$$

or equivalently that

$$s'' < \frac{(A-s)\,(1-b)(Z' + (1-b))}{Z^2} \tag{31a}$$

where we have substituted $s' = \frac{(A-s)(1-b)}{Z}$.

Differentiating s' we obtain

$$s'' = \frac{-s'(1-b)Z - Z'(A-s)(1-b)}{Z^2},$$

and thus 31a is equivalent to

$$\frac{-s'(1-b)Z - Z'(A-s)(1-b)}{Z^2} < \frac{(A-s)\,(1-b)(Z' + (1-b))}{Z^2},$$

that can be simplified to

$$-\frac{(A-s)\,(1-b)Z}{Z} < 2(A-s)Z' + (1-b).$$

Substituting $Z' = -\frac{2}{3}s'$, we obtain

$$-\,(A-s)\,(1-b) < -2(A-s)\frac{2}{3}s' + (1-b),$$

and substituting for s',

$$-\,(A-s)\,(1-b) < -2(A-s)\frac{2}{3}\frac{(A-s)\,(1-b)}{Z} + (1-b),$$

$$-\,(A-s) < -2(A-s)\frac{2}{3}\frac{(A-s)}{Z} + 1,$$

$$\frac{4}{3}\frac{(A-s)^2}{Z} - (A-s) < 1,$$

$$\frac{4\,(A-s)^2}{2(S-s)-(1-b)} - (A-s) < 1,$$

$$\frac{4\,(A-s)^2}{2(S-s)-(1-b)} < 1 + A - s,$$

which always holds since the LHS is increasing in s, the RHS decreases in s, and that the inequality holds at $s = S$.

i) Second order condition for the Stakeholder:

Similarly, second order condition for the stakeholder reduces to checking that $\frac{dW(t)}{dt} \leq 0$, where

$$W(t) = \left[\frac{5\,(S-s) - (1 + B - 2b\,(t))}{6}\right]\frac{db}{dt} - \frac{B - b(t)}{6}\left[(S-s) + (1 - b\,(t))\right].$$

We write

$$W(t) = b'U\,(t) - (B-b)V\,(t))$$

where

$$b' = \frac{(B-b)\,V}{U-V},$$

and thus the desired condition is that

$$b'' \leq \frac{(B-b)\,V' - b'\,(U' + V)}{U}, \tag{31b}$$

or equivalently that

$$\frac{b''}{B-b} \leq \frac{V' - \frac{V}{U-V}\,(U' + V)}{U}.$$

Note that

$$b'' = \frac{-b'V}{U-V} + (B-b)\left(\frac{V}{U-V}\right)' =$$

where

$$\begin{aligned}\left(\frac{V}{U-V}\right)' &= \frac{V'(U-V) - (U' - V')\,V}{(U-V)^2} = \\ \frac{-b'(U-V) - (2b' + b')\,V}{(U-V)^2} &= \frac{-b'(U-2V)}{(U-V)^2}\end{aligned}$$

$$b'' = \frac{-b'}{U-V}\left(V + (B-b)\,\frac{-(U-2V)}{(U-V)}\right) =$$

$$\frac{-b'}{U-V}\left(V + (B-b)\,\frac{-(U-2V)}{(U-V)}\right) < (B-b)\,\frac{-b' - \frac{V}{U-V}\,(2b' + V)}{U},$$

$$-\frac{VU - V^2 - BU + 2BV + bU - 2bV}{(U-V)} < (-B+b)\,\frac{U + V + \frac{V^2}{\frac{(B-b)V}{U-V}}}{U},$$

$$\frac{VU - V^2 - BU + 2BV + bU - 2bV}{U - V} > \frac{BU - bU + BV - bV + VU - V^2}{U},$$

that is equivalent to

$$\frac{(2V - U)\,(B - b)}{U - V} > \frac{(V + U)(B - b) - V(V - U)}{U}.$$

The proof is completed by noting that a sufficient condition is

$$\frac{2V - U}{U - V} > \frac{V + U}{U}$$

and this inequality always holds. ∎

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