Who Owns Children and Does It Matter?*

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Abstract

Is there an economic rationale for pronatalist policies? In this paper we propose and analyze a particular market failure that may lead to inefficiently low equilibrium fertility and therefore to a need for government intervention. The friction we investigate is related to the ownership of children. If parents have no claim on their children's income, then the private benefit from producing a child may be smaller than the social benefit. We present an overlapping-generations (OLG) model with fertility choice and altruism, and model ownership by introducing a minimum constraint on transfers from parents to children. Using the efficiency concepts proposed in Golosov, Jones, and Tertilt (2007), we find that whenever the transfer floor is binding, fertility choices are inefficient. We show how this inefficiency relates to dynamic inefficiency in standard OLG models with exogenous fertility and Millian efficiency in models with endogenous fertility. In particular, we show that the usual conditions for efficiency are no longer sufficient. Further, we analyze several government policies in this context. We find that, in contrast to settings with exogenous fertility, a PAYG social security system cannot be used to implement the efficient allocation. Rather, government transfers need to be tied to a person's fertility choice in order to provide incentives for child bearing, thus providing a justification for pronatalist policies.

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1 Introduction

In many European countries current birth rates are well below replacement levels, e.g., as low as 1.4 in Germany or 1.3 in Italy. Governments in those countries seem concerned, and are discussing, several pronatalist policies.[1] To some extent, these policies have already been implemented in various countries. For example, French parents receive generous subsidies for each child. Recently, the German government announced that it plans to triple the number of subsidized day care spots. Some Italian villages have experimented with generous one-time payments for the birth of a child.[2] Is there an economic rationale for such policies? Or should a government refrain from interfering with people's fertility choices?

In this paper we explore one particular market failure that may give rise to suboptimal fertility choice. The friction we investigate is related to ownership over children. The basic observation is that children are a resource for society. For example they increase the total labor endowment in the future. Property rights over this endowment may affect incentives. In particular, if labor income belongs to children rather than parents, then the private benefit of producing children may be smaller than the social benefit. The reason is that private benefits include only the utility from children, while the social benefits also include an increase in the aggregate time endowment. Due to this discrepancy, equilibrium fertility may be lower than what is socially optimal or efficient.[3] Even though in the formal model we focus on property rights over the labor endowment, our conclusions are more generally true. For example, if parents and children disagree about other aspects of a child's life, then who owns the right to make decisions will affect fertility choices and efficiency.

We believe that ownership of children is a relevant concept to analyze because it varies substantially across countries and has changed dramatically over time in most

[1] See for example "Europe, East and West, wrestles with falling birthrates—Long decline threatening economy," International Herald Tribune (September 3, 2006) and "Europe: The fertility bust, Charlemagne" The Economist, February 11, 2006.

[2] See "European nations offer incentives to have kids", San Francisco Chronicle, Elizabeth Bryant, August 10, 2008; "German Parents to Get More Daycare Options", Deutsche Welle, April 3, 2007. "Where have all the bambini gone?", Telegraph, April 18, 2004.

[3] Other inefficiencies relating to fertility are addressed in Pitchford (1985), Nerlove, Razin, and Sadka (1985, Oct., 1986), Lee and Miller (1990), Bruce and Waldman (1990), Harford (1998), Zhang and Zhang (2007). These concentrate on strategic considerations and a variety of externalities such as pollution.

of the developed world. Most interesting in this context are mandatory parental support or filial responsibility laws which used to make adult children legally responsible for their elderly parents. Over the course of the last two centuries, most developed countries have experienced a shift from a regime where parents had almost perfect control over their offsprings' income to a regime where children have no formal obligation towards their parents. In this paper we investigate to what extent such laws matter, from both from positive and normative perspectives—Do such laws affect the fertility decision, and what are the normative implications?

The formal model we use to analyze the importance of child ownership is an infinite horizon overlapping-generations (OLG) model with fertility choice and altruism. We formalize the idea by introducing a constraint that sets a minimal transfer from parents to children, as a fraction of a child's income. This formulation allows us to cover the full range of property rights, from parents fully owning children's labor income to a situation where children have a legal claim on their parent's income.

Analyzing normative questions in models with endogenous fertility requires taking a stand on the appropriate concept of efficiency. The reason is that Pareto efficiency is not well-defined when considering allocations with different population sizes. Of course, one can ask whether holding population size constant a Pareto-dominating allocation exists. However, such analysis yields no answer to the question whether equilibrium fertility might be inefficiently low. We therefore use an alternative concept, $\mathcal{A}$-efficiency, first proposed by Golosov, Jones, and Tertilt (2007), which is very close to Pareto efficiency but allows allocations with different population sizes to be compared.[4] In the context of models without altruism, some authors have used an alternative concept, $\mathcal{M}$-efficiency, which applies only to symmetric allocations. We focus mostly on $\mathcal{A}$-efficiency, since, analogous to Pareto efficiency, it allows people to be treated asymmetrically when constructing dominating allocations. Therefore, the set of (symmetric) $\mathcal{A}$-efficient allocations is a subset of the set of $\mathcal{M}$-efficient allocations. Appendix A.1 gives formal definitions of the concepts used here.

We find that whether or not the minimal transfer constraint is binding in equilib-

[4] All our results also go through for a second concept proposed by Golosov, Jones, and Tertilt (2007), $\mathcal{P}$-efficiency, as long as one assigns a low enough utility to unborn people.

rium depends on parameters such as the degree of altruism and the production function. Whenever the constraint is binding, fertility is inefficiently low. The finding that equilibrium fertility may be inefficiently low when property rights rest with children is interesting, because it seems to violate Coase's theorem. We argue that this inefficiency is caused by the non-existence of a market in which parents and unborn children can make trades. Such a market can never exist due to the inter-temporal nature of fertility. If parents have property rights over their children's labor income, the costs and benefits of producing new people are aligned and equilibria are generally efficient. On the other hand, if property rights are allocated to the children themselves, costs and benefits are borne by different people and inefficiency may result. Because children who are not born yet cannot negotiate and promise compensation, the original Coasian argument breaks down and the assignment of property rights becomes important for efficiency.

In standard OLG models with exogenous fertility, a necessary and sufficient condition for dynamic efficiency is that the interest rate is greater than the population growth rate (Cass (1972) and Balasko and Shell (1980)). We show that when ownership is allocated to parents (i.e., the transfer constraint is low enough), dynamic inefficiency can never occur. In this case, in line with standard results, the interest rate is bigger than the population growth rate. At the other extreme, when the minimum transfer is large enough, equilibria are dynamically inefficient in the usual sense. As in the standard case, this inefficiency is characterized by an interest rate that is below the population growth rate and a dominating allocation involves the standard transfer scheme without the need to change population. However, we show that, for intermediate values of the transfer constraint, the interest rate is above the population growth rate but the allocation is still inefficient. In this case, a dominating allocation always requires an increase in fertility. Thus, in contrast to standard OLG models, an interest rate above the population growth rate is no longer a sufficient condition for efficiency. The reason is that in addition to the potential for over-saving, when fertility is endogenous there is also a potential for under-fertility.

Another class of related models are OLG models with endogenous fertility and no altruism (e.g., Conde-Ruiz, Giménez, and Pérez-Nievas (2010) and Michel and Wigniolle (2007)). A main finding in this literature is that a sufficient condition for $\mathcal{M}$-efficiency

is that the cost of a child is higher than the present discounted value of the child's wage. We show that, while necessary, this condition is not sufficient for efficiency in our context. That is, transfer constraints may be binding in a region where the above inequality holds. In this region, it is possible to find an allocation with higher fertility that dominates the original allocation.

The existing literature has emphasized the importance of altruism for the efficiency properties of OLG models.[5] In particular, adding altruism to the standard OLG model, Barro (1974) shows that operative intergenerational transfers are a necessary and sufficient condition for efficiency. Burbidge (1983) argues that when altruism is added properly, intergenerational transfers will always be operative and the interest rate is always larger than the population growth rate, so that the potential for dynamic inefficiencies disappears. On the other hand, Pazner and Razin (1980) show that, adding fertility choice to this model, bequests are always zero, yet the allocation is still efficient.[6] Further, in a model with fertility choice but non-altruistic parents, Conde-Ruiz, Giménez, and Pérez-Nievas (2010) and Michel and Wigniolle (2007) show that an interest rate higher than the population growth rate is not a sufficient condition for (Millian) efficiency. We show in our set-up that it is not the presence or absence of altruism that leads to these differing results. Rather, the implicit assumptions on property rights are key to understanding the variety of findings. While property rights assumptions are typically not discussed explicitly, any model makes an implicit assumption about ownership. Analyzing this assumption elucidates previous findings.

Finally, we analyze the effects of various policies. We show that, if property rights lie with children and parents are therefore constrained, a PAYG pension system relaxes the transfer constraint. However, even if the pension system is such that parents are not constrained, the resulting equilibrium is not efficient.[7] The reason is that individual parents do not take into account that they are producing future contributors to the pension system. Therefore, the costs and benefits of having children are not aligned. Thus, even if taxes are lump-sum to children, they are distortionary to the fertility decision of parents. A natural way around the distortion introduced by a PAYG pension system is

[5] See, for example, Chapter 6 in Cigno and Werding (2007).
[6] Intratemporal transfers between generations, not explicitly modeled, may be non-zero however.
[7] Sinn (2004) analyzes PAYG as fertility insurance.

to make pensions a function of individual fertility choices. In fact, we show that a fertility dependent PAYG system can be used to implement an efficient allocation, which is the equilibrium allocation when parents have full property rights. This result is in line with those in Eckstein and Wolpin (1985), Abio, Mahieu, and Patxot (2004), Lang (2005) and Conde-Ruiz, Giménez, and Pérez-Nievas (2010) who use models without altruism and different efficiency/optimality concepts, but do not relate their result to the allocation of property rights. Further, we show that the same allocation can be implemented through an alternative system that subsidizes births and finances the subsidy through government debt.[8]

The idea that parents may not have access to a child's future labor income has been explored in several other contexts. In particular, several models with exogenous fertility look at the importance of this margin for education decisions. What we call property rights assigned to the child, is typically called "borrowing constraints" or "incomplete markets" in this literature. For example, Aiyagari, Greenwood, and Seshadri (2002) analyze the implications of borrowing constraints for the efficiency of investments in children in a model where fertility is exogenous. Similarly, Fernández and Rogerson (2001) analyze the implications of borrowing constraints for child schooling decisions and long-run inequality in a set-up with exogenous (but stochastic) fertility.[9] Also, Boldrin and Montes (2005) analyze a model where young adults make their own schooling decisions but are borrowing-constrained leading to an inefficiently low level of schooling. There is an important distinction, however, between the inefficiency in education and fertility choices. The cost and benefits of investing in human capital could, in principle, be borne by the same person. For example, if children made their own education investment decisions and markets are complete, then no friction exists. The same is not possible in the context of fertility decisions. It is simply not technologically feasible for a child to bear the costs of producing herself. Of course, one can design institutions that move the cost to the next generation (such as the fertility-dependent social security system we discuss in this paper), but such arrangements will always involve government intervention.

[8]For related optimal fertility policies in different setups, see Cigno (1983, 1986, 1992).
[9]See also Lazear (1983).

The remainder of the paper is organized as follows. The next section briefly discusses the existence of laws that affect ownership over children. Section 3 presents the model, characterizes equilibria and derives comparative statics. In Section 4 we analyze the efficiency properties of equilibrium fertility. Section 5 explores several government policies and Section 6 concludes.

2 Property Rights and Control over Children's Income

The argument of this paper is that the allocation of property rights over a child's income has important economic consequences. In reality these property rights differ dramatically across countries and across time. In this section, we briefly describe some of the relevant laws. We discuss evidence from both the common law system of the United States and England and with the Roman-based legal system in France.

From the perspective of our theory, we are most interested in parental control over an offspring's life-time labor income. We therefore first discuss laws that directly affect access to an offspring's labor income, such as mandatory parental support or filial responsibility laws. Laws about child labor are also relevant as they allow (or prevent) access to part of an offspring's life-time labor income. Second, we discuss laws that give parents control over other aspects of their children's lives, and thereby might allow parents to control their offsprings' income indirectly, e.g., by withdrawing consent to marriage unless monetary support is given. Third, de-facto control may vary with other aspects of society, such as living arrangements that affect the ability to monitor offsprings.

The (Elizabethan) Poor Law Act of 1601 obligated (adult) children to support their parents both in England and the U. S. (Callahan, 1985; Kline, 1992). Parents gradually lost this benefit in the mid 19th and early 20th centuries, as England and the U. S. began to repeal or ignore laws that obligated children to support their elderly parents (Thomson, 1984; Britton, 1990). Article 205 of the Napoleonic Civil code in France also specified that children had to support their elderly parents in cases of need (Byrd, 1988). This law became obsolete after the establishment of a formal social security system in the first half of the 20th century. Legal indenture of children as servants

was allowed in the U.S. throughout the 18th century (Marks, 1975). From the early 19th century, parents lost their right to indenture their children in many states, laws prohibiting children from leaving the family were "either repealed or ignored," and parents were obligated by law to provide proper care for their children (Marks, 1975).

Before the introduction of child labor and mandatory education laws, parents typically had access to their minor children's income.[10] However, by 1938, every U.S. state had passed laws which effectively banned child labor and enforced compulsory schooling (Landes and Solmon, 1972; Margolin, 1978; Guggenheim, 2005). Another important factor in a determining a parent's access to his child's resources is the legal definition of adulthood. In the 1970s, reforms were passed in the U.S. that reduced the age of majority from 21 to 18 (Castle, 1986). Around the same time, setting out the conditions under which children were released from parental authority and deemed "adults" for important legal purposes. These included rules for allowing a (minor) child to earn and spend his/her own wages (Plotkin, 1981; Davis, 2006).

Alongside the laws that directly affected parent's access over labor income, historically many laws have existed that allowed parents to control other aspects of their offsprings' lives. This control gave parents a lot of leverage over their children's resources. The most extreme example of parental control over children in the U. S. was a set of *stubborn child laws* implemented in several states during the mid-17th century, which obligated children to be obedient to their parents. If children failed to comply, parents had the right to take their children to court, and offenses could be punished by death. Corporal punishment and physical cruelty were also common methods parents used to enforce discipline (Mason, 1994). Finally, parental consent was often necessary for marriage (Kertzer and Barbagli, 2001). In contrast, there was little that children could do to protect themselves from their parents. For example a 1641 law made it illegal to curse or hit one's parents. A child who broke this law (and was over 16 years of age) could be punished by death (Hawes, 1991).

Similar degrees of parental control were also present in Roman-based legal systems (Arjava, 1998). In France, the concept of *patria potestad*, the control which a father exer-

[10]Kertzer and Barbagli (2001) point out that "for many poor parents among the working class, the artisans and the peasantry, the regular school attendance of their children implied enforced withdrawal from work, whether at home or in the workshop" which eliminated a form of income for parents.

cised over this children, underlined all legal decisions regarding children.[11] The legal system also allowed parents to use *lettres de cachet*[12] to enforce their authority when their child refused to follow parental direction regarding a marriage partner or career (Kertzer and Barbagli, 2001).

After the mid-19th century, the U. S. passed numerous laws that reduced the control of parents over their children, including strict laws on abuse, cruelty and parental neglect (Marks, 1975). Agencies, such as the *Society for the Prevention of Cruelty to Children*, were created to protect children from parental cruelty (Hawes, 1991). Finally, in the 1970s, reforms were passed that expanded medical rights for minor children (Plotkin, 1981) and greatly expanded abuse protection, such as the 1973 Child Abuse Prevention and Treatment Act (Hawes, 1991; Guggenheim, 2005). In short, by the end of the 20th century, children in the U.S. had gained rights that essentially allocated all property rights to the children themselves. Similar reforms also took place in France: "neglected children came under the protection of the courts (1889), children were protected from the physical abuse of their parents by criminal statute (1898) and 21 was established as the age of majority when children were allowed to undertake legal acts and marry without parental consent (1907)" (Kertzer 2001, p. 141).[13]

Besides these legal changes, de facto parental control over children may also have changed due to technological progress. In particular, the process of industrialization was accompanied by changing living arrangements from extended to nuclear families, which likely lowered the de facto control parents have over their children.[14] Parental control is much easier to exercise in a setting where multiple generations live together in the same household, compared to a setting where young couples live away from their parents (Folbre, 1994). Thus, the reality of living arrangements may determine de-facto property rights over children's income.

[11] Patria Potestad is "a control similar to that over material things and one which permitted a father to sell or pawn a child if necessary and even to eat it in an extreme case" (Sponsler 1982, p. 147-148).

[12] Letters signed by the king often used to enforce authority and sentence someone without trial.

[13] In legal terms, by 1972, "the relationship of parent to child [was] no longer viewed as a power of domination" and instead was "seen as an authority conferred upon parents to protect the child, thus entailing responsibilities as well as rights"(Alexandre 1972, p. 652-653).

[14] In the U. S., in the mid-19th century, almost 70 percent of persons age 65 or older resided with their adult children, while by the end of the 20th century, fewer than 15 percent did. While many scholars have attributed the change in living arrangements to increased resources of the old, Ruggles (2007) argues that increased opportunities of the young played a major role.

In sum, changes in filial responsibility laws, other parental control rights, and de-facto control all paint the same picture. Historically parents, and in most cases the father, were in a position of almost absolute control over their offsprings. Starting from the mid-19th to the mid-20th century, many different types of reforms were passed that removed most of the legal and de-facto control of parents, and essentially led offspring to gain ownership over their own earnings and their life more generally.

3 Dynastic Overlapping Generations Model with Fertility Choice

We now set up our model, and show that ownership over children matters for equilibrium fertility choices. We show this in a model where parents view children as consumption goods and are altruistic towards them. The model we use is a special case of dynastic endogenous fertility models first developed in Razin and Ben-Zion (1975). With logarithmic utility, it is also a special case of Becker and Barro (1986, 1988) and Barro and Becker (1989), though extended to two-period overlapping lives. In contrast to the existing literature, we explicitly introduce ownership over children into the model. Specifically, in the model we focus on property rights over adult children's labor income.

First, we characterize equilibria in general. Second, we solve the model for the special case of logarithmic utility and Cobb-Douglas production. We then derive comparative statics with respect to property rights and find the first order effect of a shift in property rights from parents to children is a decrease of fertility. In the next section we derive efficiency results and compare them to those in other OLG models.

3.1 Model Setup

People in our model live for three periods: childhood, (middle-aged) adulthood and retirement. In childhood, no decisions are made. Middle-aged adults work and bear children. Retired people live off their savings and potentially transfers from their chil-

dren.[15] Households derive utility from their own consumption when middle-aged, c_t^m, and when old, c_{t+1}^o, the number of children, n_t, as well as their offsprings' average utility. That is, in our model children are a consumption good in that n_t directly enters the utility function, but parents are also altruistic and care about their children's utility. The advantage of this formulation is that both models with altruism as well as models without endogenous fertility can be seen as special cases of our set-up, by setting ζ and γ to zero respectively. The utility of a middle-aged household in period t (born in $t-1$) is given by:

$$U_t = u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u(n_t) + \zeta \frac{\int_0^{n_t} U_{t+1}^i di}{n_t} \tag{1}$$

where n_t is the number of children in period t. We assume that $u(\cdot)$ is continuous, strictly increasing, strictly concave and $u'(0) = \infty$. Discounting between periods is given by β while children's (average) utility is weighted by ζ. The budget constraints are given by

$$\begin{aligned} & c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t) \\ & c_{t+1}^o + \int_0^{n_t} b_{t+1}^i w_{t+1} di \leq r_{t+1} s_{t+1} \\ & b_{t+1}^i \geq \underline{b}_{t+1} \\ & c_t^m, c_{t+1}^o, n_t \geq 0 \end{aligned} \tag{2}$$

where s_{t+1} are savings, $b_{t+1}^i w_{t+1}$ is the transfer from parent to child i if positive, from child i to the parent if negative, and θ_t is the cost per child.[16]

The minimum constraint, $\underline{b}_{t+1}$, can be interpreted as parental property rights over children's labor income.[17] When $\underline{b}_{t+1}$ is positive, then a larger transfer floor implies that parents have to bequeath more resources to their children. When $\underline{b}_{t+1}$ is nega-

[15] We introduce government transfers in Section 5.

[16] For example, $\theta_t = a_t^g + (a_t^c - \kappa_t) w_t$ with a_t^g the goods cost of children, a_t^c is the fraction of time that has to be spent with every child in raising it and κ_t is the amount of (effective) labor the parent can extract from the child. For example, if a period is 20 years and children can work from age 10 and are half as productive as an adult, then $\kappa_t \approx 0.25$. Below we concentrate on parents' property rights over adult children but a change in κ_t could reflect changes in child-labor laws, for example.

[17] Specifying transfers as absolute amounts rather than proportional to the wage leads to the same qualitative results. This is because, though chosen by the parent, both types of transfers are lump-sum to the child since labor supply in perfectly inelastic.

tive, a higher transfer floor means parents can expropriate fewer resources from their children. The transfer floor is only well-defined between -1 and some b^{max}. When $\underline{b}_{t+1} = -1$ then there are no (legal or effective) constraints on transfers and parents have full property rights over their children's income. If, on the other hand, $\underline{b}_{t+1} = 0$ then children own their own income. If $\underline{b}_{t+1} > 0$ then children have a claim to their parent's income. The maximum possible transfer, $\underline{b}_{t+1}^{max}$, is an endogenous object. At $\underline{b}_{t+1}^{max}$ a parent would bequeath his entire income to his children. A closed form expression for b^{max} as a function of parameters is derived for specific functional forms below.

Initially, there is a mass 1 of initial old people each endowed with K_0 capital and n_{-1} children. The initial old chooses $(c_0^o, \{b_0^i\}_{i=0}^{n_{-1}})$ to maximize

$$U_{-1} = \beta u(c_0^o) + \gamma u(n_{-1}) + \zeta U_0$$

subject to:

$$c_0^o + \int_0^{n_{-1}} b_0^i w_0 di \leq r_0 K_0, \quad b_0^i \geq \underline{b}_0$$

The middle-aged adult in period t chooses $(c_t^m, c_{t+1}^o, n_t, s_{t+1}, \{b_{t+1}^i\}_{i=0}^{n_t})$ to maximize U_t in equation (1) subject to the constraints in (2), given b_t, the transfer from his own parents and prices (w_t, w_{t+1}, r_{t+1}), taking the behavior of all descendants as given. Since we assume that the utility function satisfies Inada conditions, the non-negativity constraints on fertility and consumption never bind, while the minimum constraint on transfers, b_{t+1}, may or may not bind. A few additional assumptions are required to ensure that the problem is well-defined. First, we need a joint restriction on γ and ζ to rule out that the limit where $n \to 0$ and per child consumption goes to infinity yielding infinite utility for the parent. For example, with logarithmic utility,

$$\gamma > \tfrac{\zeta(1+\beta)}{1-\zeta}, \tag{3}$$

ensure that this cannot happen.[18] Another requirement for the problem to be well-defined is that the budget for the unconstrained dynastic head is not infinite. This requires that the cost of producing a child is at least as high as the (discounted) life-

[18]This is equivalent to the parameter restrictions needed in standard Barro-Becker models, in the special case of separable (logarithmic) utility. See Appendix A.2 for details.

time earnings per child: $\theta_t > \frac{w_{t+1}}{r_{t+1}}$. Without this requirement an unconstrained parent could finance infinite consumption by borrowing through his children. As we show in Section 4, this condition always holds in any unconstrained steady state. Finally, to guarantee finite utility we assume

$$\zeta < 1. \tag{4}$$

The representative firm has a neo-classical production function $Y_t = F(K_t, L_t)$, and takes prices (r_t, w_t) as given when choosing (K_t, L_t) to maximize profits. For simplicity, we assume full depreciation throughout. We assume a Cobb-Douglas production function for the special case below.

Finally, markets clear. Labor markets clear in period t if the firm's labor demand per old person, L_t, is equal to the number of middle-aged people per old person, n_{t-1}, since they are the only ones who are productive and labor is supplied inelastically. The capital stock per old person, K_t, must be equal to savings from currently old people, s_t. Denoting k_t as the capital stock per worker, we can write $s_t = k_t n_{t-1}$. Hence, factor markets clear if $L_t = n_{t-1}$ and $K_t = s_t = k_t n_{t-1}$. Goods market clearing in period t can be expressed in per old person terms as follows:

$$c_t^o + n_{t-1}(c_t^m + \theta_t n_t + s_{t+1}) = F(s_t, n_{t-1}) = n_{t-1} F(k_t, 1).$$

3.2 Characterizing equilibria

If $u(.)$ is strictly concave and there is no heterogeneity among children, it is always best for the parent to give the same transfer to each child, $b_{t+1}^i = b_{t+1}, \forall i$. Hence, we can rewrite the utility as:

$$U_t = u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u(n_t) + \zeta U_{t+1}$$

and the budget constraint when old as:

$$c_{t+1}^o + n_t b_{t+1} w_{t+1} \leq r_{t+1} s_{t+1}$$

Sequentially substituting utility functions from period s to ∞, we get:

$$U_s = \sum_{t=s}^{\infty} \zeta^{t-s} \left[u(c_t^m) + \beta u(c_{t+1}^o) + \gamma u(n_t) \right], \tag{5}$$

or, if expressed period by period and starting with the initial old:

$$U_{-1} = \sum_{t=0}^{\infty} \zeta^t \left[\beta u(c_t^o) + \zeta \{ u(c_t^m) + \gamma u(n_t) \} \right]. \tag{6}$$

Thus, the problem can be interpreted as either the middle-aged adult in period s choosing $(c_s^m, c_{s+1}^o, n_s, s_{s+1}, b_{s+1})$ to maximize U_s in equation (5) subject to the constraints in (2) for $t = s$ or, the initial old (i.e., the "Planner") making all decisions subject to the same constraints. The first-order conditions are

$$\gamma u'(n_t) = u'(c_t^m)\theta_t + \beta u'(c_{t+1}^o) b_{t+1} w_{t+1} \tag{7}$$

$$u'(c_t^m) = \beta u'(c_{t+1}^o) r_{t+1} \tag{8}$$

$$\beta u'(c_{t+1}^o) n_t = \zeta u'(c_{t+1}^m) + \frac{\lambda_{b,t+1}}{w_{t+1}} \tag{9}$$

$$c_t^m + \theta_t n_t + s_{t+1} = w_t(1 + b_t) \tag{10}$$

$$c_{t+1}^o + n_t b_{t+1} w_{t+1} = r_{t+1} s_{t+1} \tag{11}$$

$$\lambda_{b,t+1}(b_{t+1} - \underline{b}_{t+1}) = 0. \tag{12}$$

Note that, without heterogeneity other than period of birth, savings will always be positive as long as the production function satisfies Inada conditions.

The first-order conditions for the firm's problem are given by:

$$w_t = F_L(k_t, 1) \tag{13}$$

$$r_t = F_K(k_t, 1) \tag{14}$$

Combining the first-order conditions with the solution to the firm's problem, we now derive four equations that characterize the equilibrium allocation. The first two equations are intertemporal conditions equating marginal costs and benefits of investment in physical capital and children. The third condition is an *intra*temporal but *in-*

*ter*generational condition, equating the parent's marginal cost and benefit of an additional unit of transfer per child, b_{t+1}, unless the minimum constraint is binding. The fourth equation relates the equilibrium capital stock to the (binding) transfer constraint. First, using equation (14) to substitute out r_{t+1} in equation (8) gives

$$u'(c_t^m) \;=\; \beta u'(c_{t+1}^o) F_K(k_{t+1}, 1). \tag{15}$$

This is a standard Euler condition. On the left-hand side (LHS) we have the marginal cost of saving one more unit when young, while on the right-hand side (RHS) we have the marginal benefit of saving for consumption when old.

Second, combining equations (7) and (8), using equations (13) and (14), gives

$$\gamma u'(n_t) \;=\; \beta u'(c_{t+1}^o)[\theta_t F_K(k_{t+1}, 1) + b_{t+1} F_N(k_{t+1}, 1)]. \tag{16}$$

This equation equates the marginal costs and benefits from children. The marginal benefit from another child is simply $\gamma u'(n_t)$. The marginal cost consists of the forgone consumption due to the cost of producing the child, θ, and the bequest or transfer a parent wants to make to a child, $b_{t+1} F_N(k_{t+1}, 1)$. Note that the transfer might be negative, in which case the cost is lower than the child-bearing cost. Note also that since the child-bearing cost and the transfer occur in different periods, the interest rate also enters the expression.

Third, combining equations (9) and (12) gives

$$\begin{aligned} \beta u'(c_{t+1}^o) n_t \;&=\; \zeta u'(c_{t+1}^m) \qquad \text{if} \quad \lambda_{b,t+1} = 0 \\ &>\; \zeta u'(c_{t+1}^m) \qquad \text{if} \quad \lambda_{b,t+1} > 0\,. \end{aligned} \tag{17}$$

Recall that $\lambda_{b,t+1}$ is the multiplier on the transfer constraint. So when the constraint is binding, the marginal utility of consumption when old is "too high", while the marginal utility of children's consumption is "too low" from the parent's point of view. That is, consumption of the old is too low compared to their children's consumption when middle aged. This wedge is at the heart of the inefficiency.

Note that a model without altruism is a special case of our model, namely when

$\zeta = 0$. However, without altruism, parents always take everything they legally can from children. That is, the minimum transfer constraint is *always* binding so that $b_{t+1} = \underline{b}_{t+1}$.[19] On the other hand, if $\zeta > 0$ and utility, $u(\cdot)$, satisfies Inada conditions, then the parent will choose $b_{t+1} > -1$ and hence the minimal transfer constraint, $b_{t+1} \geq \underline{b}$, is not necessarily binding. For most parts of the paper we focus on the case where $\zeta > 0$, as this is the more interesting case that allows for both binding and non-binding constraints.[20]

If $\underline{b}_t = -1, \forall t$ and $\zeta > 0$, then $\lambda_{b,t} = 0, \forall t$ and we denote the equilibrium allocation by $\{c_t^{m*}, c_{t+1}^{o*}, n_t^*, s_{t+1}^*, k_t^*, b_{t+1}^*\}_{t=0}^{\infty}$ and prices by $\{w_t^*, r_t^*\}_{t=0}^{\infty}$. We denote any equilibrium allocation for the case where some generation is constrained by $\{\hat{c}_t^m, \hat{c}_{t+1}^o, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^{\infty}$ and prices by $\{\hat{w}_t, \hat{r}_t\}_{t=0}^{\infty}$.

Finally, we derive an equation that characterizes the equilibrium capital stock when parents are transfer constrained. Using equation (16) and capital market clearing in the budget constraint when old, equation (11), we get

$$\beta\theta_s \frac{F_K(\hat{k}_{s+1}, 1)}{F_L(k_{s+1}, 1)} + (\beta + \gamma)\underline{b}_{s+1} = \gamma \hat{k}_{s+1} \frac{F_K(\hat{k}_{s+1}, 1)}{F_L(k_{s+1}, 1)}. \tag{18}$$

Note that the only endogenous variable in this equation is k_{s+1}. Also note that this equation does not depend on $\underline{b}_s$; it only depends on $\underline{b}_{s+1}$ and θ_s. For given $\underline{b}_{s+1}$ and θ_s, the capital-labor ratio, $\hat{k}_{s+1}$, can be found as a solution to equation (18).

The state variable in this economy is the capital-labor ratio, K_{t+1}/n_t. Since capital depreciates fully across generations, parents are free to choose the capital labor ratio optimally, given their constraints. Therefore, if both $\underline{b}_t$ and θ_t are constant, then the economy is in steady state as of period 1, i.e., there are no transitional dynamics.

[19] Note that $\zeta = 0$ and $\underline{b} = 0$ are implicit assumptions in Eckstein and Wolpin (1985), Abio, Mahieu, and Patxot (2004), Lang (2005), Michel and Wigniolle (2007, forthcoming) and Conde-Ruiz, Giménez, and Pérez-Nievas (2010). We compare our results to theirs below.

[20] It is well known that in the standard closed economy growth model with parental altruism and fertility choice, such as the model analyzed in Barro and Becker (1989), bequests are always positive, and hence, any minimum bequest constraint less than or equal to zero, $\underline{b} \leq 0$, will never be binding. However, in our set-up, parents overlap with productive children and therefore desired bequests/transfers may well be negative. The difference is that when generations truly overlap, a negative bequest is perfectly consistent with a positive capital stock, which is not the case when each generation lives only for one period. Razin and Ben-Zion (1975) and Pazner and Razin (1980) allow for one period overlap in adult/productive lives. However, they implicitly assume that $\underline{b}_t = -1$ for all t throughout their analysis.

3.3 Analytical example

Next we explicitly derive a closed form solution for the special case of logarithmic utility together with a Cobb-Douglas production function, $F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$, with $\alpha \in (0, 1)$.

First, suppose $\underline{b}_t = -1$ for all t. Then altruism implies that no generation is constrained. In this case, the steady state capital-labor ratio, fertility and transfers are given by:

$$k^* = \frac{\alpha\theta(\beta + \zeta(1 + \beta + \gamma))}{\beta(1-\alpha) + \gamma - \alpha\zeta\,(1 + \gamma + \beta)} \tag{19}$$

$$n^* = \zeta A\alpha \left(\frac{\beta(1-\alpha) + \gamma - \alpha\zeta\,(1 + \gamma + \beta)}{\alpha\theta(\beta + \zeta(1 + \beta + \gamma))} \right)^{1-\alpha} \tag{20}$$

$$b^* = \frac{[\zeta\theta\alpha(1 + \beta + \gamma) - (1-\alpha)k^*\gamma]}{k^*(1-\alpha)\,(\gamma - \zeta\,(1 + \gamma + \beta))} \tag{21}$$

Our parameter restriction (3) guarantees that all variables are strictly positive in equilibrium. Note that the optimal transfer may well be negative. We find that b^* is negative if and only if

$$\beta(1-\alpha) > \alpha\zeta\,(1 + \gamma + \beta)\,. \tag{22}$$

To see this, note that b^* is negative if and only if

$$\theta\alpha\zeta(1 + \beta + \gamma) < (1-\alpha)k^*\gamma.$$

Using equation (19) and rearranging yields condition (22). The condition is compatible with our parameter restriction (3) as long as $\zeta < \frac{\beta}{\alpha+\beta}$, i.e., as long as parents are not too altruistic.

Condition (22) shows that parents want to take resources from children if the labor share in output is sufficiently high and if parents value their children's utility little enough relative to their own old age consumption. This shows that even altruistic parents want to take resources away from their children under certain circumstances. It also suggests that children are not only a consumption good in this model, but also an investment good.

Second, consider $\underline{b}$ such that $b^* < \underline{b}$. In this case, the parent chooses $\hat{b} = \underline{b}$ and the steady state capital-labor ratio and fertility are given by:

$$\hat{k} = \frac{\alpha\beta\theta}{\alpha\gamma - (\beta+\gamma)(1-\alpha)\underline{b}} \tag{23}$$

$$\hat{n} = \frac{\gamma A\alpha(1-\alpha)\hat{k}^{\alpha}(1+\underline{b})}{(1+\beta+\gamma)\left(\alpha\theta + \underline{b}(1-\alpha)\hat{k}\right)} \tag{24}$$

For efficiency results in Section 4, it is useful to define the following two thresholds. Let $\underline{b}_P$ be the transfer constraint such that $\hat{n} = \hat{r}$ and let $\underline{b}_M$ be the transfer constraint such that $\hat{w} = \theta\hat{r}$. Using the equations above, we can derive closed form solutions for $\underline{b}_P$ and $\underline{b}_M$:

$$\underline{b}_P = \frac{\alpha(1+2\beta+\gamma) - \beta}{(1-\alpha)(1+2\beta+\gamma)} \tag{25}$$

$$\underline{b}_M = \frac{\gamma\alpha - \beta(1-\alpha)}{(1-\alpha)(\beta+\gamma)} \tag{26}$$

Now, from the solution for $\hat{k}$, the maximal $\underline{b}$ for which a steady state equilibrium exists is $\underline{b}^{max} = \frac{\gamma\alpha}{(1-\alpha)(\beta+\gamma)}$. It is straightforward to see that $\underline{b}_P < \underline{b}^{max}$ if and only if $(1+2\beta+\gamma)\alpha < 1$. Since this conditions does not contradict the parameter restrictions needed for the model to be well defined—conditions (3) and (4)— a low enough α is sufficient to guarantee the existence of $\underline{b}_P$. Clearly, $\underline{b}_M < \underline{b}^{max}$ is always true for admissible parameters.

3.4 Changes in property rights and fertility choice

Here we analyze how a permanent shift in property rights (from parents to children) affects fertility choice. A tightening of the transfer constraint, if binding, means that children are more costly, so that the first order effect is a decline in fertility. However, several indirect effects are present and complicate the analysis. Except for those who are old when the law is changed, a tightening of the transfer constraint affects people in two ways. First, they receive less from their own children, but they also owe less to their own parents. The latter is an income effect which may lead to higher desired

fertility. Second, general equilibrium effects are present as the tightening of the transfer constraint affects the capital labor ratio, thus changing relative prices, which in turn changes the incentive to invest in children vs. capital. Note that the exact timing of the legal change matters. A change in the constraint starting from period $s+1$ that is announced in s has different effects than does a surprise change in $s+1$. We now disentangle these direct and indirect effects more formally.

Assume the economy is in a steady state where the constraint is constant and binding at $\underline{b}$. In period s, there is an (unanticipated) permanent shift in property rights from parents to children so that $\underline{b}' > \underline{b}$ in all future periods. We assume that this change in the law takes place *after* transfers of generation s to their own parents have taken place, but before any other decisions are made. In other words, $\underline{b}_s = \underline{b}$ while $\underline{b}_t = \underline{b}' > \underline{b}$ for all $t > s$. Therefore, the change in the law affects generation s differently from generations born later. With this thought experiment in mind, we use marginal arguments below to simplify the algebra. For this analysis, it is useful to derive the general expression for equilibrium fertility (i.e., the non-steady state analog of (24)). For the logarithmic utility case, the expression is

$$n_t = \frac{\gamma}{1+\beta+\gamma}\left(\frac{w_t(1+\underline{b}_t)}{\theta_t + \underline{b}_{t+1}\frac{w_{t+1}}{r_{t+1}}}\right) \tag{27}$$

First, consider generation s. Note that $\underline{b}_s$ is unchanged by assumption and that w_s is only a function of the capital-labor ratio which was chosen in the previous period and is therefore unaffected by the legal change. Therefore, the change in fertility is entirely determined by the change in the ratio $\frac{\underline{b}_{s+1}w_{s+1}}{r_{s+1}}$. From equation (18), it follows that

$$\frac{\underline{b}_{s+1}w_{t+1}}{r_{s+1}} = \frac{\gamma k_{s+1} - \beta\theta_s}{\beta+\gamma}. \tag{28}$$

Combining equations (27) and (28) and taking the total derivative, holding $\underline{b}_s$ and w_s fixed, we have

$$\frac{dn_s}{d\underline{b}_{s+1}} = -\frac{\beta+\gamma}{1+\beta+\gamma}\left(\frac{w_s(1+\underline{b}_s)}{(\theta_s+k_{s+1})^2}\right)\frac{dk_{s+1}}{d\underline{b}_{s+1}} \tag{29}$$

Therefore, to determine the effect of the legal change on n_s we only need to determine the change in the capital-labor ratio in period $s+1$. The sign of $\frac{dk_{s+1}}{d\underline{b}_{s+1}}$ is positive

as long as capital and labor are substitutable enough (see Appendix A.3 for details). For example, this is the case with a Cobb-Douglas production function. It follows from (29) that the fertility effect on generation s is unambiguously negative for this case. To sum up, for generation s, the effective cost of children, $(\theta_s + \underline{b}_{s+1} w_{s+1}/r_{s+1})$, increases in $\underline{b}_{s+1}$ as long as labor and capital are substitutable enough because this guarantees an increase in w_{s+1}/r_{s+1}.

Second, the effects on later generations are more complicated. Taking into account the relationship between $\frac{\underline{b}_{t+1} w_{t+1}}{r_{t+1}}$ and k_{t+1} as described above, we can write equilibrium fertility for any generation $t > s$ as

$$n_t = \frac{\beta + \gamma}{1 + \beta + \gamma} \left(\frac{w_t(\underline{b}')(1 + \underline{b}')}{\theta_t + k_{t+1}(\underline{b}')} \right) \tag{30}$$

Taking the total derivative, we have

$$\begin{aligned} \frac{dn_t}{d\underline{b}} = \quad & -\frac{\beta + \gamma}{1 + \beta + \gamma} \left(\frac{w_t(1 + \underline{b})}{(\theta_t + k_{t+1})^2} \right) \frac{dk_{t+1}}{d\underline{b}} \\ & + \frac{\beta + \gamma}{1 + \beta + \gamma} \left(\frac{w_t}{\theta_t + k_{t+1}} \right) + \frac{\beta + \gamma}{1 + \beta + \gamma} \left(\frac{1 + \underline{b}}{\theta_t + k_{t+1}} \right) \frac{dw_t}{d\underline{b}} \end{aligned} \tag{31}$$

Comparing equation (31) to (29) we see that for generations beyond s there are two additional terms—both dampening the fertility decline. The first term comes from the fact that these later generations owe their own parents less, which is a positive income effect, leading to higher fertility. The second term is a general equilibrium effect. Their own wage rate increases in response to the change in $\underline{b}$ (because of the increase in the capital stock) which generates a further positive income effect.

The overall effect of the change in property rights on fertility for generations $t > s$ may therefore be positive or negative. However, in most examples, there is a range of large $\underline{b}$ where an increase in $\underline{b}$ causes fertility to fall, i.e., the total derivative is negative.

4 Property Rights and Efficiency

In this section we analyze the efficiency properties of equilibria in our model. Analyzing normative questions in models with endogenous fertility requires taking a stand on the appropriate concept of efficiency. The problem is that Pareto efficiency is not a well-defined concept when considering allocations with different population sizes. One might still ask whether a given allocation is Pareto efficient, i.e., whether for a given population a dominating allocation exists. However, this kind of analysis cannot address the question whether equilibrium fertility is too low. We use an alternative concept, $\mathcal{A}$-efficiency, first proposed by Golosov, Jones, and Tertilt (2007), which is very close to Pareto efficiency but allows us to compare allocations with different population sizes.[21] In the context of models without altruism, some authors have used an alternative concept, Millian efficiency ($\mathcal{M}$-efficiency), which requires potentially dominating allocations to be symmetric across all people within a given generation. Note that the set of (symmetric) $\mathcal{A}$-efficient allocations is a subset of the set of $\mathcal{M}$-efficient allocations. In other words, by widening the set of potentially dominating allocations, we can identify inefficiencies that cannot be addressed if symmetry is imposed. Appendix A.1 gives formal definitions of all concepts.

Using these concepts, we compare our findings to the previous literature on efficiency in OLG models. We first state two basic results that characterize when equilibria are efficient. The analysis thereafter includes both a comparison with dynamic efficiency in standard OLG models and one with $\mathcal{M}$-efficiency in OLG models with endogenous fertility but no altruism. We show how previous results about dynamic and $\mathcal{M}$-efficiency depend on whether the transfer constraint is binding or not. We also show that sufficient conditions for efficiency derived in the literature no longer apply when considering $\mathcal{A}$-efficiency in a model with altruism.

[21] All our results also go through for a second concept proposed by Golosov, Jones, and Tertilt (2007), $\mathcal{P}$-efficiency, as long as one assigns a low enough utility to unborn people.

4.1 $\mathcal{A}$- and $\mathcal{P}$-efficiency of competitive equilibrium allocations

Our first result states that equilibria in an economy without transfer constraints are always $\mathcal{A}$-efficient.

Proposition 1 *Assume $\zeta > 0$. If $\underline{b}_t = -1$ for all t, then the equilibrium allocation, $\{c_t^{m*}, c_{t+1}^{o*}, n_t^*, s_{t+1}^*, k_t^*, b_{t+1}^*\}_{t=0}^{\infty}$, is $\mathcal{A}$- (and $\mathcal{P}$-) efficient.*

Proof. This follows from Theorem 2 in Golosov, Jones, and Tertilt (2007). Without transfer constraints, the equilibrium allocation maximizes the utility of the dynastic head, given prices. Thus, the equilibrium allocation is dynastically $\mathcal{A}$- (and $\mathcal{P}$-) efficient. This, together with the assumption of a neoclassical production function, ensures that the assumptions of Theorem 2 in Golosov, Jones, and Tertilt (2007) are satisfied. ■

On the other hand, when there are binding constraints, then the equilibrium allocation is always $\mathcal{A}$-inefficient, as the next proposition shows.

Proposition 2 *Assume $\zeta > 0$. If $\lambda_{b,s+1} > 0$ for some generation s, then the equilibrium allocation, $\{\hat{c}_t^m, \hat{c}_{t+1}^o, \hat{n}_t, \hat{s}_{t+1}, \hat{k}_t, \hat{b}_{t+1}\}_{t=0}^{\infty}$, is $\mathcal{A}$- (and $\mathcal{P}$-) inefficient.*

Proof. We prove this by constructing an alternative allocation that $\mathcal{A}$-dominates the equilibrium. The alternative allocation has an $\varepsilon > 0$ amount of additional children for each member of generation s. Each new child transfers an additional $\delta > 0$ resources to her parent. This new allocation is feasible and for small ε and δ, this new allocation makes generation s strictly better off, and no one worse off. See Appendix A.4 for a formal proof. ■

It is worth noting that the unconstrained equilibrium allocation, though $\mathcal{A}-$efficient, is not necessarily $\mathcal{A}-$superior to the equilibrium allocation when the constraint is binding. This is because, apart from the initial old, every subsequent generation may be worse off.

Finally, let us consider the special case of no altruism. Without altruism, parents do not value their children's consumption and hence the transfer constraint is always binding. As long as the legal constraint $\underline{b}$ is not at the feasible minimum, this means that such an equilibrium is not $\mathcal{A}$-efficient. The logic is the same as in the proof of

Proposition 2. The logic breaks down if the legal constraint coincides with the feasible minimum, $\underline{b} = -1$. For this special case, the equilibrium is both $\mathcal{A}$-efficient and the constraint is binding. Note, however, that such an equilibrium is very strange: the initial old expropriate all income from their children, who consequently consume zero, and no children are born. Clearly, the only stationary equilibrium for this case is trivial: no one is alive. We summarize these results in the next proposition.

Proposition 3 *Assume $\zeta = 0$. Then the transfer constraint is always binding. There are two cases:*
a) if $\underline{b} > -1$, then the equilibrium is $\mathcal{A}$- (and $\mathcal{P}$-) inefficient.
b) if $\underline{b} = -1$, then the equilibrium is such that $c_t^m = c_{t+1}^o = n_{t-1} = 0$ for all $t \geq 1$, and the equilibrium is $\mathcal{A}$- (and $\mathcal{P}-$) efficient.

4.2 Property Rights vs. Altruism

Our set-up differs from the basic OLG literature along two dimensions: parents are altruistic and fertility is endogenous. Table 1 classifies the previous literature along these two dimensions. We find that differences in results in the previous literature are mainly due to (implicit) assumptions about property rights, rather than to the presence (or absence) of altruism, as is often stated.

Table 1: Literature Comparison

	exogenous fertility	endogenous fertility
without altruism	Samuelson (1958), Cass (1972), Balasko and Shell (1980)	Eckstein and Wolpin (1985), Conde-Ruiz, Giménez, and Pérez-Nievas (2010), Michel and Wigniolle (2007, forthcoming)
with altruism	Barro (1974), Burbidge (1983)	Pazner and Razin (1980), This Paper

The potential for dynamic inefficiencies in OLG models has long been recognized, going back to Samuelson (1958). Several papers have added altruism to OLG models with exogenous fertility (e.g., Barro (1974)). A result is that an OLG economy with

altruism behaves like an infinitely lived agent economy and hence dynamic inefficiencies disappear. For example, Burbidge (1983) showed that when altruism is properly added to the standard OLG model, then the interest rate will *always* be larger than the population growth rate, and hence the equilibrium allocation will always be Pareto efficient.[22] This result is derived in the endogenous fertility context by Pazner and Razin (1980), who also find that equilibrium allocations are always dynamically efficient in the sense that $r > n$.[23]

Our Proposition 2 shows that altruism is perfectly consistent with inefficiencies occurring in equilibrium. In other words, it is not the presence or absence of altruism alone that is the dividing line between equilibrium efficiency and inefficiency. Rather, inefficiencies occur precisely when the transfer constraint is binding, i.e., when parents have too few property rights relative to their degree of altruism. Previous results implicitly relied on the assumption that transfer constraints are not binding. More specifically, the equivalence between an OLG model with altruism and an infinitely lived consumer model, as pointed out by Barro (1974), holds only if resources can be transferred freely across generations—at least intra-temporally. However, this means that parents are allowed to take as much as they want from their children.[24] There are several reasons why models with altruism typically abstract from transfer constraints. First, models without constraints are easier to analyze. Second, once altruism is introduced it might appear natural to let a dynastic head make all the decisions for the dynasty. However, another natural benchmark is that children have full rights to their own (labor) endowments. In this case, inefficiencies may well occur in equilibrium, even in models with altruism. Note that the result that full parental rights are sufficient to guarantee efficiency is a very general one. It holds independent of whether altruism

[22]For example, in Burbidge (1983)'s setting, $r > \bar{n}$ always holds. His setting has no fertility choice (i.e., $\gamma = \theta = 0$). Altruism is added by defining the intergenerational discount factor as $\zeta = \bar{n}/(1+\rho) < 1$, where ρ is the intergenerational rate of time preference, $\bar{n}$ the exogenously given population growth rate. It then follows that $r = (1+\rho)$ in equilibrium and, since $\zeta < 1$, the result follows. There was a heated debate about these issues at the end of the 1970's and early 1980's. See for example, Drazen (1978), Carmichael (1982), Buiter and Carmichael (1984), Burbidge (1984), Abel (1987) and Laitner (1988).

[23]Pazner and Razin (1980) is the only previous paper that has used the expression "property rights" in this context. However, they analyze only the case where parents have full property rights.

[24]The usual interpretation is that parents and children share the same preferences whenever they are alive. Hence children would willingly make these transfers. Such a specification is isomorphic to ours where parents have all property rights and transfer constraints are not binding.

is assumed, and also independent of whether fertility is exogenous or endogenous.

Recently, several authors have analyzed OLG models with endogenous fertility, but without altruism. This literature has pointed out that once fertility is endogenous, standard results in the OLG literature fail to hold. In particular, Conde-Ruiz, Giménez, and Pérez-Nievas (2010) and Michel and Wigniolle (2007, forthcoming) give examples of economies where equilibria are not $\mathcal{M}$-efficient. Again, it is often pointed out that the inefficiency can be attributed to the absence of altruism.[25] However, our Proposition 3 shows that the crucial assumption that leads to equilibrium $\mathcal{A}$-inefficiency is not the lack of altruism, but rather the assignment of property rights to children. In fact, essentially all models without altruism assume that children have full property rights over themselves. This assumption is natural in this context. In models with exogenous fertility and no altruism, parent-child relationships are not even clearly defined and hence the natural starting point is self-ownership for each agent in the economy. Once fertility choice is added there are well-defined family relationships. However, as long as altruism is absent, parents will always take everything they legally or feasibly can from their children. Thus, as shown in Proposition 3, case (b), not imposing any transfer constraints implies that only parents consume anything, children starve and the economy ends thereafter—not a very interesting case. Hence, such models typically assume $\underline{b} = 0$.

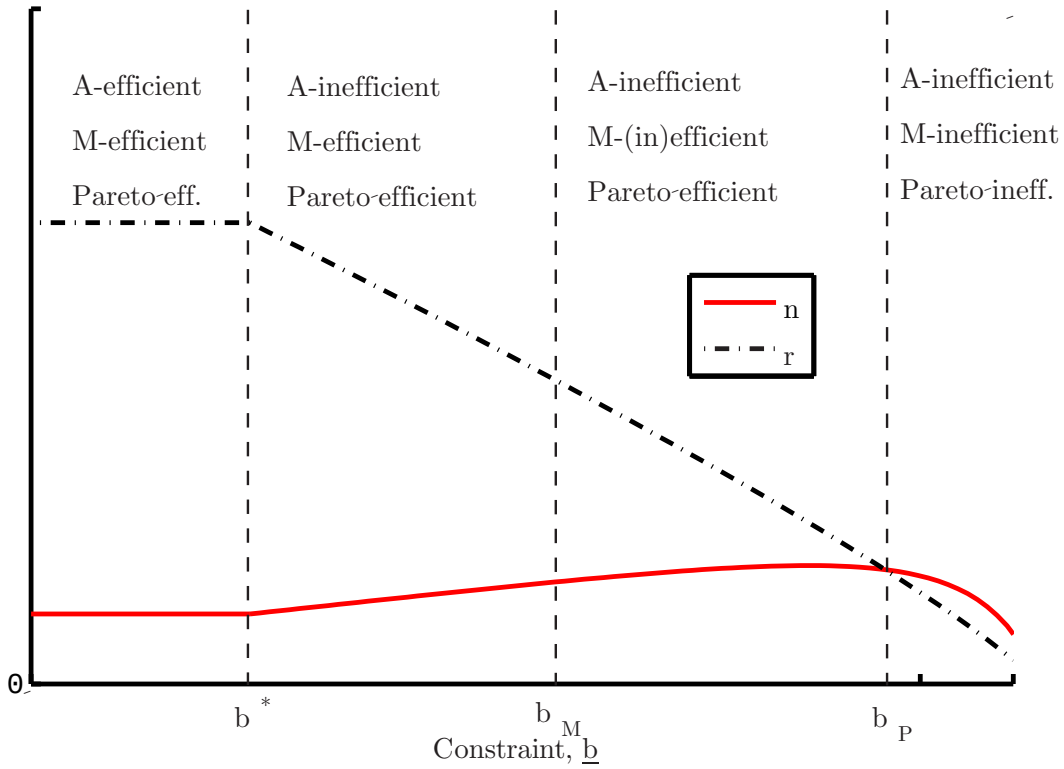
4.3 Necessary and Sufficient Conditions for Efficiency

We now derive necessary and sufficient conditions for efficiency. Figure 1 shows a stylized description of how steady state interest and fertility rates change as a function of $\underline{b}$. The picture shows three distinct cases. When $\underline{b}$ is low, the constraint is not binding (case 1). Second, when the constraint starts to bind, the interest rate is still above the population growth rate at first (case 2). The dividing line between cases 1 and 2 is simply the equilibrium transfer chosen when the constraint is not binding, b^*. As $\underline{b}$ increases further, at some point the interest and fertility rates cross. Thus, our third case consists of all $\underline{b}$ where the interest rate is below population growth (case 3). The dividing line is b_P, defined to be the constraint for which in equilibrium $n = r$. This

[25] For example, Cigno and Werding (2007) point to this dichotomy (p.121 and p.125).

threshold relates our results to the OLG literature with exogenous fertility. Further, to relate our results to the literature on efficiency in OLG models with endogenous fertility, we use a third threshold, b_M, to be such that in equilibrium $w = \theta r$. This threshold typically lies within case 2. We explain below what assumptions guarantee the existence of these cut-offs. In fact, for the Cobb-Douglas case, closed form solutions for b^*, b_P and b_M exist (see Section 3.3).

Figure 1: Steady State Characterization as a Function of $\underline{b}$



4.3.1 Case 1 $[\underline{b} < b^*]$

For low enough minimum transfers from parents to children, the constraint is not binding. This is because with altruism, parents want their children to consume something. In this case, equilibria are $\mathcal{A}$-efficient. This is the result in Proposition 1.

We know from Golosov, Jones, and Tertilt (2007) that $\mathcal{A}$-efficiency implies Pareto-efficiency (when fertility is held constant), hence dynamic efficiency is guaranteed in this case. Therefore the allocation in case 1 is also Pareto efficient. In standard OLG models (first developed by Samuelson (1958) and Diamond (1965)), the stationary equilibrium allocation is dynamically efficient if and only if $r > n$ (e.g., Cass (1972) and Balasko and Shell (1980)). The analogous condition in our set-up is different.

Proposition 4 *A stationary equilibrium allocation is $\mathcal{A}$- (and $\mathcal{P}$-)efficient if and only if*

$$n = \zeta r$$

Proof. This result follows directly from equations (8) and (9), together with Propositions 1 and 2 that state that the equilibrium is inefficient if and only if the constraint is binding. ■

Recall that $\zeta < 1$ is necessary for the model to be well-defined. Thus, Proposition 4 immediately implies that any $\mathcal{A}$-efficient allocation is characterized by $r > n$. However, as we will see in case 2, the condition is not sufficient here.

4.3.2 Case 2 $[\underline{b} \in (b^*, b_P)]$

As soon as $\underline{b} > b^*$, the constraint binds. Once the constraint binds, the equilibrium allocation is $\mathcal{A}$-inefficient as Proposition 2 showed.

Note that case 2 is still characterized by $r > n$. Hence, an immediate implication is that, in contrast to models with exogenous fertility, $r > n$ is not a sufficient condition for $\mathcal{A}$-efficiency. The reason is that over-saving is not the only potential problem in this economy. Instead, child-bearing may be too low in equilibrium. We summarize these insights as a proposition.

Proposition 5 *In a stationary equilibrium, $r > n$ is a necessary but not a sufficient condition for $\mathcal{A}$-efficiency.*

Proof. Necessity follows from Proposition 4 and condition (4). To show that it is not sufficient, realize that for $\underline{b} = b^*$ we have $r > n$ by Proposition 4. By continuity, there exists $\underline{b} > b^*$ such that $r > n$. This together with Proposition 2 proves the result. ■

Thus for any $\underline{b} \in (b^*, b_P)$, the equilibrium allocation is $\mathcal{A}$-inefficient but Pareto efficient. The result that $r > n$ is not a sufficient condition for $\mathcal{A}$-efficiency may have important policy implications. Sometimes the $r > n$ criterion is used to assess whether a particular country is dynamically efficient (e.g., Abel, Mankiw, Summers, and Zeckhauser (1989)). This can be relevant in the context of designing social security systems, for example. Our findings suggest that such analysis may have been based on the wrong criterion—given that, in the developed world at least, by and large people do choose their own birth rate.

Several authors have analyzed models with endogenous fertility but without altruism (see Conde-Ruiz, Giménez, and Pérez-Nievas (2010) and Michel and Wigniolle (2007, forthcoming)). Proposition 3 shows that interior equilibria can never be $\mathcal{A}$-efficient when parents are not altruistic. The reason is that without altruism (in our model: $\zeta = 0$), *any* transfer constraint binds since parents optimally extract the maximum feasible amount from their children. Papers without altruism therefore typically use a different efficiency concept: $\mathcal{M}$-efficiency, which is similar to $\mathcal{A}$-efficiency but requires people within a generation to be treated symmetrically. Generally speaking more allocations are $\mathcal{M}$-efficient than $\mathcal{A}$-efficient, because fewer allocations are considered for potential domination. In particular, only allocations that treat everyone in a given generation identically are considered, whereas in our proof of $\mathcal{A}$—inefficiency, we constructed a superior allocation that treated new people differently from those who are alive under both allocations.

These authors also find that $r > n$ is not sufficient for $\mathcal{M}$-efficiency. Instead, they find that a sufficient condition for $\mathcal{M}$-efficiency is given by $r\theta > w$ (see Conde-Ruiz, Giménez, and Pérez-Nievas (2010), Proposition 5 and Corollary 2, and Michel and Wigniolle (2007), Proposition 4). Again, we find that $r\theta \geq w$ is necessary, but not sufficient for $\mathcal{A}$-efficiency. In fact, for many utility specifications an even stronger result is true: $\mathcal{A}$-efficiency implies that $r\theta > w$ holds with strict inequality. See Appendix A.2 for details.

Proposition 6 *In a stationary equilibrium, $r\theta \geq w$ is a necessary but not sufficient condition for $\mathcal{A}$-efficiency.*

Proof. Necessity is shown by way of contradiction. Assume z^* is an interior $\mathcal{A}$-efficient

stationary equilibrium allocation for which $r\theta < w$ holds. From Propositions 1 and 2 we know that z^* is the solution to the problem when the constraint is not binding. Note that the budget constraints can be combined into a single budget constraint as follows:

$$c_t^m + \frac{c_{t+1}^o}{r_{t+1}} = w_t(1+b_t) + \left\{ \frac{-b_{t+1}w_{t+1}}{r_{t+1}} - \theta_t \right\} n_t.$$

However, since $r\theta < w$, there exists $b_{t+1} > -1$ such that $\left\{ \frac{-b_{t+1}w_{t+1}}{r_{t+1}} - \theta_t \right\} > 0$. By choosing this transfer, it is affordable to produce an arbitrarily large amount of children. Since children would be left with an endowment of $w_{t+1}(1+b_{t+1}) > 0$, their utility would be finite, while the utility from number of children would be infinite. Hence the utility of generation t would be infinite and z^* cannot be optimal. This is a contradiction. It proves that $r\theta \geq w$ is a necessary condition for $\mathcal{A}$-efficiency. To show that the condition is not sufficient, we provide a counterexample. Suppose $\underline{b} = 0$, production is Cobb-Douglas and utility is logarithmic. Then the constraint is binding if and only if

$$\beta(1-\alpha) > \alpha\zeta(1+\beta+\gamma).$$

Using the functional form assumptions, $\hat{w} < \theta\hat{r}$ reduces to $\beta(1-\alpha) < \alpha\gamma/\theta$. Choosing α small enough guarantees that the first inequality holds. Then, one can choose θ small enough to guarantee that the second inequality holds. Thus, we constructed an example where the constraint is binding and by Proposition 2 the allocation is inefficient, yet, $w < \theta r$. ■

The proposition implies that for b^* between b^* and $\underline{b}_M$) (i.e., case 2a) the equilibrium allocation is $\mathcal{A}$-inefficient but $\mathcal{M}$-efficient. If capital and labor are substitutable enough in production, we have shown in Section 3.4 that w increases in $\underline{b}$, while r decreases. Hence, as $\underline{b}$ increases further, $w > \theta r$. It seems plausible therefore that a range of $\underline{b}$ exists within $(\underline{b}_M, \underline{b}_P)$ where equilibrium allocation are $\mathcal{M}$-inefficient but still Pareto efficient (case 2b). Michel and Wigniolle (2007, forthcoming) implicitly assume $\underline{b} = 0$ but derive interesting results in this context. Proposition 3 in Michel and Wigniolle (forthcoming) implies that with a logarithmic utility and Cobb-Douglas production function, equilibria are $\mathcal{M}$-efficient if and only if they are Pareto efficient (see also Remark 5 in

Michel and Wigniolle (2007)). Further, they show that a more general production function with an elasticity of substitution greater than one may indeed lead to equilibrium allocations that are Pareto efficient, but $\mathcal{M}$-inefficient (see Proposition 3 in Michel and Wigniolle (2007)). Such an equilibrium is also $\mathcal{A}$-inefficient. While the dominating allocation (with unequal treatment) used in the proof of Proposition 2 still works, in this case, one can also find dominating allocations that do not involve asymmetries between people within the same generation.

4.3.3 Case 3 $[\underline{b} \in (\underline{b}_P, \underline{b}^{max})]$

In this case $r < n$, and hence the equilibrium is $\mathcal{A}$-, $\mathcal{M}$- and Pareto-inefficient. In this case people are saving too much, and a dominating allocation can be constructed by redistributing resources across generations (holding population size fixed).

Does this case always exist? Not necessarily. Depending on functional forms and parameters, this Pareto-inefficient case may never be reached. The reason is that there is a maximally feasible level of $\underline{b}$. Once a parent is required to transfer resources to the child that exceed the income of the parent, the economy is no longer well-defined. The question thus becomes whether the interest rate becomes smaller than the fertility rate before this maximal level is reached, or not. For the Cobb-Douglas case, closed form solutions for $\underline{b}_P$ and $\underline{b}^{max}$ exist (see Section 3.3). The following proposition follows immediately.

Proposition 7 *With Cobb-Douglas production and log utility, Pareto inefficiency occurs for high enough $\underline{b}$ if and only if $\alpha < \frac{1}{1+2\beta+\gamma}$.*

Thus, models with endogenous fertility and altruism can be inefficient in the usual sense, i.e., dynamically inefficient, as long as the capital share is low and people care enough about old age consumption (high β) and the number of children (high γ). If the capital share is high relative to these utility parameters, then accumulating capital is productive enough so that over-accumulation does not occur.

5 Policy Implications

Given the equilibrium inefficiencies resulting from binding transfer constraints, the most obvious policy recommendation would be to simply lift the constraints and give parents full property rights over their children. However, such a policy might not be desirable for various reasons. For example, some parents may abuse their children. Further, there will always be some people who simply cannot have children for medical reasons. Most importantly, it might be very difficult to enforce payments from adult children to their parents. While all these additional concerns are outside of our model, we still believe it useful to explore to what extent alternative policies can also implement efficient allocations in equilibrium.

For example, a pay-as-you-go (PAYG) pension system essentially provides a way of transferring resources from the young to the old. Hence, a PAYG system may be desirable in societies where children have rights over their labor income. On the other hand, it may further distort the incentives to have children. In addition to a standard PAYG system, we also examine a fertility dependent PAYG pension system as well as fertility subsidies financed with government debt. In each case, we ask whether a given policy allows the implementation of $\mathcal{A}$-efficient allocations.

5.1 PAYG social security

We introduce a pay-as-you-go social security system (PAYG) in the model laid out in Section 3. First, we show that the introduction of a standard PAYG social security system, in which children are taxed to finance lump-sum transfers to parents when old, alleviates the downward pressure on fertility and increases the desired transfer when parents are constrained. Second, we show that the standard PAYG system cannot be used to implement an $\mathcal{A}-$efficient allocation. The intuition for this result is that, while taxes that finance pension payments may be lump-sum to children, they are distortionary to the parent's fertility decision.

The government taxes middle aged people at rate τ_t and gives the proceeds as a lump-sum pension, T_t, to the old. Both the children and parents take these taxes and

pensions as given. Hence, the modified budget constraints are:

$$c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t - \tau_t) \tag{32}$$

$$c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1} s_{t+1} + T_{t+1} \tag{33}$$

To simplify algebra, we specify taxes proportional to wages. Note, however, that labor is supplied inelastically, and therefore our specification, is equivalent to lump-sum taxes for generation t.

A PAYG system requires the government to balance its budget every period. Hence, we have $T_{t+1} = n_t \tau_{t+1} w_{t+1}$. That is, the government chooses one instrument, say τ_{t+1}, while the other, T_{t+1}, is determined in equilibrium, by the fertility choice of all parents. The (infinitesimal) individual parent realizes that his/her fertility choice alone will not affect the average pension and hence takes T_{t+1} as given. Otherwise, everything in this set-up is the same as before. In particular, other than the budget constraints none of the first-order conditions of the household or the firm and none of the feasibility conditions are affected by this change.

First, assume that $\underline{b}$ is high enough so that the transfer constraint is binding. Using the first-order condition in equation (16), the budget constraint when old, capital market clearing and the government's budget balance, one can derive the analog of equation (18) for $\tau_{s+1} > 0$:

$$\beta\theta_s \frac{F_K(\hat{k}_{s+1}, 1)}{F_N(\hat{k}_{s+1}, 1)} + (\beta + \gamma)\underline{b}_{s+1} - \gamma\tau_{s+1} = \gamma \frac{F_K(\hat{k}_{s+1}, 1)}{F_N(\hat{k}_{s+1}, 1)} \hat{k}_{s+1}. \tag{34}$$

The only difference from equation (18) is the term $\gamma\tau_{s+1}$. Again, assume that capital and labor are substitutable enough, then the capital-labor ratio is decreasing in τ_{s+1} (see Appendix A.3 for details). Using this, one can derive the analog of equation (30):

$$\hat{n}_s = \frac{\gamma(w_s(1 + \underline{b}_s - \tau_s))}{\gamma k_{s+1} + (1 + \gamma)\theta_s + \underline{b}_{s+1} \frac{w_{s+1}}{r_{s+1}}}. \tag{35}$$

Comparative statics with respect to τ are very similar to those in Section 3.4. We concentrate here on the direct effect of an increase in τ. Specifically, assume τ_{s+1} increases,

but τ_s stays the same. Thus, generation s's tax contribution is not affected, but generation s knows that they will receive higher pensions when old. By construction, this change does not affect the capital labor ratio in period s and therefore w_s is also constant. Since $\frac{d\hat{k}_{s+1}}{d\tau_{s+1}} < 0$ so that $\frac{w_{s+1}}{r_{s+1}}$ unambiguously decreases, it follows that $\frac{d\hat{n}_s}{d\tau_{s+1}} > 0$. Thus, if generation s is initially transfer constrained, the introduction of a standard PAYG social security system increases fertility. The reason is a simple income effect from increased pension income. [26]

An increase in τ also eventually relaxes the transfer constraint. To see this, recall that if $\lambda_{s+1} > 0$, from equation (17) we have

$$\beta u'(c^o_{t+1})n_t > \zeta u'(c^m_{t+1}).$$

Using the budget constraints in equations (32) and (33), the introduction of a PAYG pension system clearly tends to increase the RHS and decrease the LHS of this inequality. Thus, for a large enough tax system the transfer constraint is no longer binding. For example, if $\tau_{t+1} = w_{t+1}(1 + \underline{b}_{t+1})$, the parent would like to make a transfer $b_{t+1} > \underline{b}_{t+1}$ so that the children's consumption is positive.

Even though transfers can become operative if the PAYG tax is large enough (i.e., the constraint may become irrelevant), the resulting equilibrium is nevertheless not $\mathcal{A}-$efficient. To see this combine the budget constraints in equations (32) and (33) to get

$$c^o_{t+1} + n_t(c^m_{t+1} + \theta_t n_{t+1} + s_{t+2} - w_{t+1} + \tau_{t+1}w_{t+1}) \leq r_{t+1}s_{t+1} + T_{t+1}$$

It is immediately apparent that the "lump-sum" tax on children, τ_{t+1}, is distortionary to the parent: the more children he/she has, the more taxes his/her dynasty pays. That is, parents do not internalize that children are future contributors to the social security system, T_{t+1}, and therefore do not produce the efficient number of children.[27]

[26] Subsequent generations also experience a negative income effect because their tax burden increases and wages decrease (see the numerator of equation (35)). Hence their fertility may increase or decrease. See also Nishimura and Zhang (1995) and Boldrin, De Nardi, and Jones (2005) for quantitative analyses of the effects of PAYG social security on fertility in models with endogenous fertility.

[27] See Boldrin, De Nardi, and Jones (2005), p. 40, for a similar point.

This result is in contrast with the exogenous fertility dynastic OLG literature, starting with Barro (1974) and followed by Carmichael (1982), Burbidge (1983), Abel (1987) and others, where operative bequests or transfers are a sufficient condition for optimality or Pareto efficiency. The basic problem with a standard PAYG system is that the costs and benefits of producing children remain unaligned.

5.2 Fertility dependent PAYG pensions

The obvious way to align the cost and benefits of having children is to make the pension system fertility dependent (FDPAYG), the focus of this section.[28] We show how such a system, like standard PAYG, alleviates the downward pressure on fertility caused by binding property rights. Since parents are altruistic in our setup, FDPAYG also generates an increase in the desired transfer. If the FDPAYG system is large enough, the allocation of consumption levels is the same as in the case where parents have full property rights. Thus FDPAYG can be used to implement an $\mathcal{A}$-efficient allocation. Interestingly, in the spirit of this result, several countries have now made provisions for time spent raising children to count towards pension entitlements. In France, for example, a child supplement of 10% is added to social security benefits if the person raised at least three children.[29]

As before, the government taxes the middle aged at rate τ_t and gives the proceeds as a fertility dependent pension, $T_t(n_{t-1})$, to the old. That is, the parent knows that an increase in her own fertility affects her pension payment when old. Hence, the budget constraints now are:

$$c_t^m + \theta_t n_t + s_{t+1} \leq w_t(1 + b_t - \tau_t)$$
$$c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1} s_{t+1} + T_{t+1}(n_t)$$

Again, a PAYG system requires that the government balances its budget:

[28] Eckstein and Wolpin (1985), Abio, Mahieu, and Patxot (2004), Lang (2005) and Conde-Ruiz, Giménez, and Pérez-Nievas (2010) also point out that a fertility-dependent social security system is optimal. In contrast to our analysis, their results are derived in a model without altruism. Moreover, as mentioned before, the optimality concepts used differ from ours. Finally, property rights are assumed to lie with children throughout their analysis.

[29] Many other European countries have similar provisions, see Social Security Administration (2004).

$T_{t+1}(n_t) = n_t \tau_{t+1} w_{t+1}$.

For the household, this change alters the first-order condition for fertility, n_t. Assuming pensions are linear functions of fertility so that $T'_{t+1}(n_t) = \tau_{t+1} w_{t+1}$, condition (16) becomes:

$$\gamma u'(n_t) \quad = \quad \beta u'(c^o_{t+1})[F_K(k_{t+1}, 1)\theta_t + (b_{t+1} - \tau_{t+1})F_N(k_{t+1}, 1)]. \tag{36}$$

Assume that $\underline{b}$ is high enough, so that, given τ, the transfer constraint is binding. Using the first-order condition in equation (15), the budget constraints for the household and the government's budget balance, one can derive the analog of equation (18) for $\tau_{s+1} > 0$:

$$\beta\theta_s \frac{F_K(\hat{k}_{s+1}, 1)}{F_N(\hat{k}_{s+1}, 1)} + (\beta + \gamma)(\underline{b}_{s+1} - \tau_{s+1}) = \gamma \frac{F_K(\hat{k}_{s+1}, 1)}{F_N(\hat{k}_{s+1}, 1)} \hat{k}_{s+1}. \tag{37}$$

This equation is very similar to (34). Note, however, that now $\underline{b}$ and τ enter in exactly the same way. Therefore, simply by setting $\tau = \underline{b}$, we can mimic a world without transfer constraints and without a pension system.

One can also derive the analog of equation (30) for $\tau_{s+1} > 0$:

$$\hat{n}_s = \frac{\beta + \gamma}{1 + \beta + \gamma} \left(\frac{\hat{w}_s(1 + \underline{b}_s - \tau_s)}{\theta_s + \hat{k}_{s+1}} \right) \tag{38}$$

Again, we consider the simple case where τ_t changes for all t starting at $t \geq s+1$ and is unchanged prior to that. This means that $\hat{w}_s$, $\underline{b}_s$ and τ_s are all fixed. Assuming that capital and labor are substitutable enough, it follows that $\frac{d\hat{n}_s}{d\tau_{s+1}} > 0$ since $\frac{d\hat{k}_{s+1}}{d\tau_{s+1}} = -\frac{d\hat{k}_{s+1}}{d\underline{b}_{s+1}} < 0$ (see Appendix A.3 for details). Thus, if generation s is constrained, the introduction of a fertility dependent PAYG social security system leads to an increase in fertility. It also relaxes the transfer constraint in a way similar to the standard PAYG. Contrary to the standard PAYG system however, a large enough FDPAYG system implements an $\mathcal{A}-$efficient allocation, because by choosing a large enough τ the government can undo the effect of the binding transfer constraint. Rather than parents taking from their own children, the government taxes all children and then allocates funds to the old taking the number of children into account.

Finally, note that if the policy goal is to implement this particular $\mathcal{A}-$efficient allocation (i.e., the one that coincides with the equilibrium allocation in a world where parents have full property rights), there is no unique "optimal tax", but an entire range of large enough FDPAYG taxes that implement the same $\mathcal{A}-$efficient allocation. If τ is set larger than $\underline{b}w$, parents will simply undo the "too large" pension payment by giving transfers back to their own children. Note that this result is different from Eckstein and Wolpin (1985), Abio, Mahieu, and Patxot (2004), Lang (2005) and Conde-Ruiz, Giménez, and Pérez-Nievas (2010) who all find a unique optimal fertility dependent tax level in related contexts but without altruism.[30]

This result speaks to the current policy debate that blames low fertility rates for the insolvency of the standard PAYG systems around the western world. While a social security system may have seemed the obvious solution to old age poverty in a world where children were no longer obliged to look after their parents, it should have been designed to avoid further distortion of fertility decisions.

5.3 Fertility subsidies and government debt

Another pronatalist policy that is seen to varying degrees in many countries are fertility subsidies. For example, many countries have tax deductions for children. Some countries also give a one-time subsidy for the birth of each child. For example, the Russian government pays 4,500 Rubles for the birth of each child. Similarly, several cantons in Switzerland and some cities in Italy pay large birth grants.[31] We now show that in the context of our model, fertility subsidies make sense. They give an incentive to increase child-bearing and, if set at a high enough level, can lead to efficient fertility choices. In particular, we show that the unconstrained equilibrium allocation can be implemented through a policy that subsidizes fertility and finances these subsidies by issuing debt. The debt is then repaid by taxing the next generation, i.e., the children, in a lump-sum fashion a period later.

Let τ_t^s be the per child subsidy a parent receives and τ_t^d a labor income tax rate on

[30] It is important to note that the allocation resulting from a large enough FDPAYG system is not the only efficient allocation. It is also not $\mathcal{A}-$superior to the allocation where parents are constrained and taxes are zero because, except for the initial old, every subsequent generation may be worse off.

[31] See Social Security Administration (2004) for details of such policies in European countries.

all young people. Let d_{t+1} be per middle-aged person debt issued by the government.

$$c_t^m + \theta_t n_t + s_{t+1} + d_{t+1} \leq w_t(1 + b_t - \tau_t^d) + \tau_t^s n_t$$
$$c_{t+1}^o + b_{t+1} w_{t+1} n_t \leq r_{t+1}(s_{t+1} + d_{t+1})$$

Government budget balance (per old person) requires that

$$n_{t-1}(d_{t+1} + \tau_t^d w_t) = r_t d_t + \tau_t^s n_t n_{t-1}$$

holds in all periods.

Now suppose the government sets $\tau_t^d = \tau_t$ and $\tau_t^s = \tau_{t+1}\frac{w_{t+1}}{r_{t+1}}$ where τ_t are the taxes specified for the FDPAYG pension above. It is straightforward to see that the household's budget set in period t is the same as for the FDPAYG pension. Further, if the government issues debt $d_{t+1} = \tau_t^s n_t$, then the government budget constraint holds every period. Therefore, for large enough fertility subsidies, the $\mathcal{A}$-efficient allocation can be implemented as an equilibrium outcome.

In sum, fertility subsidies together with taxes on the next generation to finance the these subsidies is identical, in our model, to allowing parents to leave negative bequests to their own children. In a more complicated model the two policies might not be exactly identical. In fact, fertility subsidies might be more desirable. For example in a world with uncertainty about the type (e.g., labor productivity) of one's own children, a fertility subsidy effectively offers insurance against low quality children. Such insurance is not offered by simply allowing parents to tap into their own children's income.

6 Conclusion

In this paper we analyze the effects of various degrees of parental control over children's labor income. We start by documenting that in most developed countries, laws implemented in the last two centuries effectively reallocated property rights from parents to children. First we characterize equilibria in an OLG model with endogenous fertility where parents are altruistic towards children. We show that when children

own themselves, the costs and benefits of having children are not aligned, which can lead to inefficiently low fertility. Furthermore, we show that property rights are relevant in reconciling results from models with and without altruism, and with and without endogenous fertility. We also show how property rights over children interact with other intergenerational policies. We show that a standard PAYG system will not lead to an $\mathcal{A}$-efficient allocation because even though taxes when middle-aged are lump-sum to children, they are distortionary from the parent's point of view. We therefore examine alternative pension systems, in particular one where pension payments are a function of fertility choices, as well as fertility subsidies and government debt. Both systems are able to implement an $\mathcal{A}$-efficient allocation. These policies might be problematic, of course, when fertility has a stochastic component. In this case, the system may need to make provisions for involuntary infertility.

The focus of our work so far is on the theoretical properties of fertility models and the characterization of equilibria with and without parental control rights. Alongside the theory, the ideas presented here may have something to say about the historical fertility experience in the United States. A combination of shifts in property rights and the introduction of PAYG social security may help account for fertility patterns over the past two centuries, including the baby boom and bust. This combination may also have generated Caldwell (1978)'s reversal of net transfers between parents and children. That is, a combination of laws (restrictions on transfers together with a social security system) lead from a situation where transfers run from children to parents, to a world where transfers run from parents to children. Whether this channel indeed played a quantitatively important role in U.S. fertility history will be analyzed in future research.

The set-up could be easily extended to analyze many other interesting applications. For example, constraints on transfers are likely to be binding only for some families. Introducing heterogeneity and analyzing the impact of legal changes on differential fertility would be very interesting. Some authors such as De la Croix and Doepke (2003, 2004, 2009), Doepke (2004) and Zhao (2009) have used ability or "skill" heterogeneity to generate differential fertility. Jones, Schoonbroodt, and Tertilt (2008) give examples of preference heterogeneity. Drazen (1978), Laitner (1979) and Cukierman

and Meltzer (1989) determine which types of households are likely to be bequest-constrained in models with exogenous fertility. The analysis of differential fertility and property rights would require us to derive similar comparative statics in models with endogenous fertility.

Also, it might be fruitful to explicitly model the distinction between transfers from minor vs. adult children to parents. For example, child labor laws were concerned with minor children, while the English poor laws were concerned with adult children. Introducing this distinction into the theoretical set-up may shed further light on the importance of such laws. Interactions between fertility and educational choice may also be interesting in this context.

Finally, since labor supply is essentially inelastic in our setup and all agents are identical, it does not matter whether taxes are lump-sum or proportional to wages. In a setting with a labor-leisure choice, human capital investments, heterogeneity or even private information,[32] this equivalence no longer holds. The analysis of policies that lead to $\mathcal{A}$-efficient allocations in these contexts is useful for policy makers.

In this paper, we take the shift in property rights as given and explore its consequences. Yet, a big open question is *why* laws shifting property rights from parents to children were introduced. At least two potential answers come to mind. One would be that legal constraints shifted for political economy reasons (e.g., that a majority of people voted for children's rights due to increased longevity, for example). Alternatively, the reason behind changes in de-facto ownership may have been driven by technological changes. For example, the change from an agricultural rural society to an industrialized urban society may have brought a change in the de facto control parents have over their children. We leave this investigation to future research.

[32] see Hosseini, Jones, and Shourideh (2009) for example

A Appendix

A.1 Formal definitions of Efficiency

Here we formally define the four notions of efficiency that we use throughout the paper. We begin with the usual notion of Pareto efficiency for completeness. Key here is that population size is fixed in any potentially dominating allocation. To deal with endogenous fertility, Golosov, Jones, and Tertilt (2007) suggest two generalizations of Pareto efficiency that allow for changing sets of people when comparing alternative allocations: $\mathcal{A}$- and $\mathcal{P}$-Efficiency. We also define Millian efficiency, a concept used in several other papers. This concept is similar to $\mathcal{A}$- efficiency, but differs in that it requires potentially dominating allocations to be symmetric within generations.

Let z denote an allocation, i.e., $z = \{c_t^m, c_{t+1}^o, n_t, s_{t+1}, k_t, b_{t+1}\}_{t=0}^{\infty}$. Throughout the paper we also assume that $u_i(unborn) < u_i(z)$ for all z in which i is born.

Definition 8 *A feasible allocation $z = \{z_i\}_i$ is Pareto efficient if there is no other feasible allocation $\hat{z}$ with the same set of people alive such that*

1. $u_i(\hat{z}) \geq u_i(z) \ \ \forall i$.
2. $u_i(\hat{z}) > u_i(z)$ *for some* i.

Let $\mathcal{P}$ be the set of potential people. An allocation $z = \{z_i\}_{i \in \mathcal{P}}$ specifies z_i the vector of all the goods over which i's utility is defined. Let A be the set of all possible allocations. Further, let $A(i)$ be the set of all allocations in which i is born.

To define $\mathcal{A}-$efficiency, the following assumption is needed:

Assumption 9 *For each $i \in \mathcal{P}$, there is a well defined, real-valued utility function* $u_i : A(i) \to \mathbb{R}$.

Definition 10 *A feasible allocation $z = \{z_i\}_i$ is $\mathcal{A}$-efficient if there is no other feasible allocation $\hat{z}$ such that*

1. $u_i(\hat{z}) \geq u_i(z) \ \ \forall i$ *alive in both allocations.*
2. $u_i(\hat{z}) > u_i(z)$ *for some i alive in both allocations.*

To define $\mathcal{P}-$efficiency, the following assumption is needed:

Assumption 11 *For each $i \in \mathcal{P}$, there is a well defined, real-valued utility function $u_i : A \rightarrow \mathbb{R}$.*

Definition 12 *A feasible allocation $z = \{z_i\}_i$ is $\mathcal{P}$-efficient if there is no other feasible allocation $\hat{z}$ such that*

1. *$u_i(\hat{z}) \geq u_i(z)$ for all $i \in \mathcal{P}$*
2. *$u_i(\hat{z}) > u_i(z))$ for at least one $i \in \mathcal{P}$.*

Definition 13 *A feasible allocation $z = \{z_i\}_i$ is $\mathcal{M}$-efficient if there is no other feasible allocation $\hat{z}$ such that*

1. *$u_i(\hat{z}) \geq u_i(z) \ \ \forall i$ alive in both allocations.*
2. *$u_i(\hat{z}) > u_i(z)$ for some i alive in both allocations.*
3. *$u_i^t(\hat{z}) = u_j^t(\hat{z}) \ \ \forall i, j$ alive in both allocations, where t indexes the generation.*

A.2 Non-separable preferences and the logarithmic special case

Suppose that, instead of separable preferences as considered in the main text, preferences are of the Barro-Becker type and given by

$$U_t = u(c_t^m) + \beta u(c_{t+1}^o) + \zeta g(n_t) U_{t+1}$$

Assume that and $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $g(n) = n^{1-\varepsilon}$. Then, following Jones and Schoonbroodt (forthcoming), there are three sets of parameter restrictions that ensure that utility satisfies the usual monotonicity and concavity properties:

AI. $[0 < \varepsilon < \sigma < 1]$. In this case, number and utility of children are complements in the utility of the parent and the intertemporal elasticity of substitution in consumption (IES) is larger than one.

AII. $[1 < \sigma < \varepsilon]$. In this case, number and utility of children are substitutes in the utility of the parent and the IES is smaller than one.

AIII. [$1-\varepsilon = \delta(1-\sigma)$ with $\delta > 1$ and $\sigma \to 1$]. In this case, utility is separable and logarithmic.[33] This case is identical to our formulation (5) with logarithmic utility when $\gamma \equiv \frac{\delta\zeta(1+\beta)}{1-\zeta}$. Since $\delta > 1$, it follows immediately that $\gamma < \frac{\zeta(1+\beta)}{1-\zeta}$, which is our condition (3).

It follows that a general separable utility formulation is not a special case of the Barro-Becker preferences, except for the logarithmic case. This limits the scope and ease of direct comparisons with results in non-altruistic models with endogenous fertility. Because of this, we chose a general separable utility formulation as our main specification. Nevertheless, with minor restrictions on the additional parameters, similar results to those derived in Section 4 go through also with the non-separable Barro-Becker preferences. In particular:

- Propositions 1 and 2 remain identical.
- Similar to Proposition 4, a stationary equilibrium allocation is $\mathcal{A}$- (and $\mathcal{P}$-)efficient if and only if $n^{\varepsilon} = \zeta r$.
- Similar to Proposition 5, it follows that if ε is large enough, then $r > n$ is necessary for $\mathcal{A}$-efficiency. In particular, $\varepsilon \geq 1$ is large enough, which is guaranteed in cases AII. and AIII. That $r > n$ is not sufficient still follows by continuity.
- Regarding Proposition 6, a stronger result holds, namely $\theta r > w$ (with strict inequality) is necessary for $\mathcal{A}$-efficiency.[34] Indeed, consumption when middle aged in an unconstrained stationary equilibrium is given by

$$c^{m*} = \left(\frac{1-\sigma}{\sigma-\varepsilon}\right)\left(\frac{\theta - w^*/r^*}{r^* + (\beta r^*)^{\frac{1}{\sigma}}}\right).$$

  Since $\left(\frac{1-\sigma}{\sigma-\varepsilon}\right) > 0$ under any parameter configuration, $\theta r^* > w^*$ must hold at an interior unconstrained equilibrium and necessity for $\mathcal{A}$-efficiency follows as before. That $\theta r > w$ is not sufficient follows by continuity.

[33] Details on the necessary utility transformations that lead to this result are available upon request.

[34] Note that we rule out the case where $\sigma = \varepsilon$ in all three parameter configurations. In this case, only aggregate consumption enters the Planner's utility and population becomes a standard investment good. The arbitrage condition holds with equality, $\theta r = w$. See Jones and Schoonbroodt (forthcoming) for details.

In sum, if number and utility of children are not too complementary in the utility of the parent, or equivalently, if the intertemporal elasticity of substitution is low enough, all our results go through. This restriction is not very stringent. In particular, standard values for the intertemporal elasticity of substitution in the growth and business cycle literature, as well as in the recent fertility literature, fall within this range.

A.3 Comparative statics of the capital-labor ratio

Suppose $F(K_t, L_t) = A\left[\alpha K_t^{\rho} + (1-\alpha)L_t^{\rho}\right]^{\frac{1}{\rho}}$. In this case, we have

$w_t = F_L(k_t, 1) = A(1-\alpha)\left[\alpha k_t^{\rho} + (1-\alpha)\right]^{\frac{1-\rho}{\rho}}$

$r_t = F_K(k_t, 1) = A\alpha\left[\alpha + (1-\alpha)k_t^{-\rho}\right]^{\frac{1-\rho}{\rho}} = A\alpha\left[\alpha k_t^{\rho} + (1-\alpha)\right]^{\frac{1-\rho}{\rho}} k_t^{\rho-1}$

where $k_t \equiv \frac{K_t}{L_t}$. Using this in equation (18), we get

$$\beta\theta_s\left(\frac{\alpha}{1-\alpha}\right)k_{t+1}^{\rho-1} + (\beta+\gamma)\underline{b}_{t+1} = \gamma\left(\frac{\alpha}{1-\alpha}\right)\hat{k}_{t+1}^{\rho}.$$

Since $\rho < 1$, the LHS of this equation is always decreasing in $\hat{k}_{t+1}$. The RHS is

- increasing in $\hat{k}_{t+1}$ if $\rho > 0$ (substitutes case);
- independent of $\hat{k}_{t+1}$ if $\rho = 0$ (the Cobb-Douglas Case);
- decreasing in $\hat{k}_{t+1}$ if $\rho < 0$ (complements case).

Now, an increase in $\underline{b}_{t+1}$ shifts the LHS up. As long as the RHS is either weakly increasing ($\rho \geq 0$) or decreasing but less steep than the LHS ($\gamma\rho\hat{k}_{t+1} > -\beta\theta_t(1-\rho)$), we have $\frac{d\hat{k}_{t+1}}{d\underline{b}_{t+1}} > 0$. To see this formally, take the total derivative to get

$$\frac{d\hat{k}_{t+1}}{d\underline{b}_{t+1}} = \frac{\beta+\gamma}{\hat{k}_{t+1}^{\rho-2}\left(\frac{\alpha}{1-\alpha}\right)\left(\gamma\rho\hat{k}_{t+1} + \beta\theta_t(1-\rho)\right)}$$

Thus $\frac{d\hat{k}_{t+1}}{d\underline{b}_{t+1}} > 0$ if and only if

$$\gamma\rho\hat{k}_{t+1} > -\beta\theta_t(1-\rho). \tag{39}$$

Similarly, we can show that under PAYG,

$$\frac{d\hat{k}_{t+1}}{d\tau_{t+1}} = -\frac{\gamma}{\hat{k}_{t+1}^{\rho-2}\left(\frac{\alpha}{1-\alpha}\right)\Big(\gamma\rho\hat{k}_{t+1} + \beta\theta_t(1-\rho)\Big)}.$$

Thus, $\frac{d\hat{k}_{t+1}}{d\tau_{t+1}} < 0$ if and only if condition (39) holds. Finally, we can show that under FDPAYG,

$$\frac{d\hat{k}_{t+1}}{d\tau_{t+1}} = -\frac{\beta+\gamma}{\hat{k}_{t+1}^{\rho-2}\left(\frac{\alpha}{1-\alpha}\right)\Big(\gamma\rho\hat{k}_{t+1} + \beta\theta_t(1-\rho)\Big)} = -\frac{d\hat{k}_{t+1}}{d\underline{b}_{t+1}}.$$

Thus, $\frac{d\hat{k}_{t+1}}{d\tau_{t+1}} < 0$ if and only if condition (39) holds. This implies that in both cases $\frac{dw_{t+1}}{d\tau_{t+1}} > 0$ and $\frac{dr_{t+1}}{d\tau_{t+1}} < 0$.

A.4 Proof of Proposition 2

Proof. Consider the following alternative allocation. Generation s is allocated the following consumption, savings and number of children:

$$\begin{aligned} \tilde{c}_s^m &= \hat{c}_s^m - \theta_s\varepsilon & \tilde{n}_s &= \hat{n}_s + \varepsilon \\ \tilde{c}_{s+1}^o &= \hat{c}_{s+1}^o + (\delta - \bar{b}_{s+1}\hat{w}_{s+1})\varepsilon & \tilde{s}_{s+1} &= \hat{s}_{s+1}. \end{aligned}$$

The ε mass of newborn children (who are adults in period $s+1$) receive:

$$\begin{aligned} \tilde{c}_n^m &= \frac{F(\hat{s}_{s+1}, \tilde{n}_s) - F(\hat{s}_{s+1}, \hat{n}_s)}{\varepsilon} - \hat{s}_{s+2} - \theta_{s+1}\hat{n}_{s+1} + \bar{b} - \delta \\ \tilde{c}_n^o &= \hat{c}_{s+2}^o \qquad \tilde{n}_n = \hat{n}_{s+1} \qquad \tilde{s}_n = \hat{s}_{s+2} \end{aligned}$$

That is, we are giving the newborns an equal fraction of the extra output they produce and take $\delta - \bar{b}_{s+1}\hat{w}_{s+1}$ away from each child when middle-aged to give to the parent when old in period $s+1$—that is, they give δ more to their parents than do their siblings. They do, however, have the same fertility, savings, and consumption when old as their siblings. While everyone else receives the same as in the hat equilibrium allocation.

That is $\forall t \neq s$:

$$\tilde{c}_t^m = \hat{c}_t^m \qquad \tilde{n}_t = \hat{n}_t$$
$$\tilde{c}_{t+1}^o = \hat{c}_{t+1}^o \qquad \tilde{s}_{t+1} = \hat{s}_{t+1}$$

Note that feasibility of the alternative allocation follows directly by construction. Essentially the additional output that is produced by having more workers in period $s+1$ is divided equally among the additional children, minus the net transfer ($\delta - \bar{b}_{s+1}\hat{w}_{s+1}$) they give to their parents. We now show that, for small ε and δ, the alternative allocation is $\mathcal{A}$-superior to the equilibrium allocation. To do this, define $\tilde{U}_t$ to be the utility of generation t under the new allocation and $\hat{U}_t$ under the equilibrium allocation respectively. Then, it is easy to see that $\tilde{U}_t = \hat{U}_t$ for all $t > s$. Further, for the ε mass of new people, we have:

$$\tilde{U}_n(\varepsilon, \delta) = \hat{U}_{s+1} - u(\hat{c}_{s+1}^m) + u(\tilde{c}_n^m)$$

For generation s (i.e., the parents of the new children), we have:

$$\tilde{U}_s(\epsilon, \delta) = u(\tilde{c}_s^m) + \beta u(\tilde{c}_{s+1}^o) + \gamma u(\tilde{n}_s) + \zeta \left(\frac{\hat{n}_s \tilde{U}_{s+1} + \epsilon \tilde{U}_n(\epsilon, \delta)}{\hat{n}_s + \epsilon} \right)$$

Using the construction of the tilde allocation, this is equal to

$$u(\hat{c}_s^m - \theta\epsilon) + \beta u(\hat{c}_{s+1}^o + \epsilon(\delta - \bar{b}_{s+1}\hat{w}_{s+1})) + \gamma u(\hat{n}_s + \epsilon) + \zeta \left(\frac{\epsilon[u(\tilde{c}_n^m) - u(\hat{c}_{s+1}^m)]}{\hat{n}_s + \epsilon} \right) + \zeta \hat{U}_{s+1}$$

Taking the derivative with respect to ϵ and evaluating the expression at $\epsilon = 0$, we have

$$\frac{\partial \tilde{U}_s(\epsilon, \delta)}{\partial \epsilon}\Big|_{\epsilon=0} = -\theta u'(\hat{c}_s^m) + \beta u'(\hat{c}_{s+1}^o)[\delta - \bar{b}_{s+1}\hat{w}_{s+1}] + \gamma u'(\hat{n}_s) + \zeta \frac{[u(\hat{c}_{s+1}^m - \delta) - u(\hat{c}_{s+1}^m)]}{\hat{n}_s}$$

Using equation (7), this reduces to

$$\beta u'(\hat{c}_{s+1}^o)\delta + \frac{\zeta}{\hat{n}_s}[u(\hat{c}_{s+1}^m - \delta) - u(\hat{c}_{s+1}^m)]$$

Note that for $\delta = 0$, this term is zero. So all that is left to show is that for a small

increase in δ, this term increases. Taking derivatives with respect to δ, we have:

$$\frac{\frac{\partial \tilde{U}_s(\epsilon,\delta)}{\partial \epsilon}\big|_{\epsilon=0}}{\partial \delta}\Big|_{\delta=0} = \beta u'(\hat{c}^o_{s+1}) - \frac{\zeta}{\hat{n}_s} u'(\hat{c}^m_{s+1})$$

Which is equal to $\frac{\lambda_b}{\hat{w}_{s+1}\hat{n}_s}$ by equation 9, which is strictly positive if and only if the constraint is binding. Hence, for small ϵ and δ, generation s is strictly better off with the alternative allocation. Finally, any generation prior to s has generation s as a descendant, and hence is also strictly better off. This completes the proof that the alternative allocation $\mathcal{A}$-dominates the equilibrium allocation. If in addition we also have $u(F_L(\hat{K}, \hat{n}) - \hat{s} - \theta\hat{n} + \underline{b}) > u(unborn)$ then the new children are also strictly better off, and hence the alternative allocation also $\mathcal{P}$-dominates the equilibrium allocation. ■

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