Teaching-Research and Design Experiment

Two Methodologies of Integrating Research and Classroom Practice

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The paper discusses the relationship between two approaches to the integration of research with teaching practice, the Design Experiment and Teaching-Research. The Design Experiment approach is based on the idea of transferring the psychological laboratory of the educational researcher directly into the mathematics classroom, so that the complexity of the synergistic life of the classroom is directly under the observation and control of the researcher. Teaching-Research (TR-NYC model) is the approach developed in urban community colleges of New York City, which allows for the application of research into classroom practice by the mathematics instructor and for the process of deriving and verifying scientific hypotheses out of classroom practice. TR-NYC model, a non-hierarchical approach to teaching-research, empowers the practitioner, cultivating the process of inquiry and adding personal meaning to the act of teaching. It is most suited to the present situation when reform is needed in most of the classrooms of mathematics. Two examples of the classroom research are provided, one from practice to educational knowledge base, and another from the research results to classroom practice.

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The paper reviews the Teaching–Research and the Design Experiment methodologies – two reform oriented approaches to teaching and research, which have acquired new importance and significance [Kelly and Lesh, 2000]. Teaching-Research, the process of developing the craft of teaching through the adoption and assimilation of scientific methodology in classroom work, originates in the principles of Action Research, and its developmental history has been documented by several authors [King and Lonnquist, 1992; Noffke,1994]. Its original principles were formulated by [Stenhouse,1975]. Teaching-Research is credited with the increase of teachers' classroom awareness, significant increase of work motivation as well as the increase of student achievement and interest in the subject matter. It allows the teacher to view the classroom as the scientific laboratory focused on the improvement of the process of learning. Design Experiment methodology in education, on the other hand, has its origins in Vygotskian notion of the Teaching Experiment performed "to study mental changes under the effect of instruction", according to [Menchinskaya,1967; Hunting, 1983]. The "scientific status" of Design Science was delineated by [Simon, 1970], while the Design Experiment as a contemporary tool of educational research was formulated by [Brown, 1992] and [Collins,1992]. Both approaches, Teaching-Research and Design Experiment share important characteristics of contributing to building the bridge between the educational research and teaching practice. We examine the common features as well as the most important differences of both methodologies; we propose a new model of teaching-research formulated recently in the community colleges of CUNY in New York City (TR-NYC model), which integrates their differences [Czarnocha, 2002].

Gap between Educational Research and Teaching Practice

The document How People Learn [NRC,1999], asserts that one of the fundamental difficulties in the progress of reform in mathematics and science education is the lack of integration between research and teaching. This shortcoming results not only in a time lag between theory and practice but also reinforces the widespread belief that researchers' endeavors are irrelevant to classroom practices [Brown 1992; Saul 1995]. What is the conceptual content of the divide and how does it arise?

To begin with, the teacher and researcher each have different interests in the classroom. On one hand, [Cobb and Steffe, 1983] assert that the interest of a researcher during the teaching experiment in the classroom is "in hypothesizing what the child *might* learn and finding [as a teacher] ways and means of fostering that learning". On the other hand, the interest of a Teacher-Researcher is to formulate ways and means to foster *what a child needs to learn* in order to reach a particular moment of discovery or to master a particular concept of the curriculum [Czarnocha, 1999]. Since, however, "such moments occur only within students' autonomous cognitive structures, the [constructivist] teacher has to investigate these structures during a particular instructional sequence [in order to be of help to the students]. In this capacity, he or she acts as a researcher".

Different interests dictate different research questions arriving from different sources and reaching different aims. For Anne Brown, one of the educational researchers whose work significantly brought the educational laboratory much closer to the classroom [Brown, 1992], the goals were "…to engineer interventions which not only work by recognizable standards but are also based on theoretical descriptions that delineate why they work, and thus render them reliable and repeatable." (p.143) The source of her ideas was primarily in the theoretical descriptions, in a theoretical framework. For a teacher, on the other hand, the source of ideas is in solving pragmatic didactic problems in the classroom, which with time leads to the rich, if intuitive, teaching knowledge. The very recognition of that knowledge, the professional content knowledge as an independent entity [Shulman, 1986] is only 18 years old. However, the means of determining what is this knowledge, what are its components, and what is the proper conceptual framework, are not yet entirely clear. Yet it is these riches of teachers' knowledge, their natural spontaneous questions arising out of practice, intuitive hints and their confirmations that are the sources of classroom research. This significant difference in nature and source of research ideas, if not coordinated, can cause "professional bypassing", an example of which is described by [Wiske,1995]: "The teachers defined the issue in terms of the types of problems, taken from their textbooks, tests, workbooks that students frequently failed. The professor defined the issue in terms of an underlying mathematical concept, described in language that was unfamiliar to most teachers…The professor recalls the early conversations with teachers…as full of conflict. He and teachers became polarized over the way they defined the important questions worth investigating…".

What is urgently needed here is a systematic way of translating, coordinating or unifying the language between the two professional communities. In a forthcoming paper the Schema Theory and Teaching-Research in Mathematics we argue that the concept of a schema can be such a unifying tool. In the context of a schema, the characterization through "the type of problems", and characterization in terms of "the underlying

mathematical concepts" are but two different stages in the development of the same concept. Characterization by "the type of the problems" indicates the second (Inter) stage of the relevant conceptual schema [Piaget and Garcia,1989], when the learner sees the relationships between different individual cases (between different problems); while characterization by the underlying mathematical concept, is the quality of the third (Trans) stage of schema development, when out of the network of relationships, a new concept arises which reorganizes the whole network [Einstein, 1949]. Thus if, instead of seeing the two conceptions (teachers' and researchers' approaches) as exclusive and opposing, we understand them as two different stages of knowledge, both in its own right, both needed to be mastered, then we can start moving in between them, for teachers to acquire the researcher Trans stage, while for researchers to understand the underlying mathematical concept in terms of the problem structure of the second, Inter stage of the schema.[1]

The ease with which one theoretically can approach the cognitive gap between the two components of the profession is contrasted by the strength with which the social gap between them can be reinforced. There are many authors who point out to diverse aspects of the absence of equality between the teacher and researcher communities [e.g. Bishop, 1998; Newman et al, 1989; Wiske, 1995]. However, one of the most precise characterization of this socio/professional gap can be found in the words of [Wittman, 1998] who, in his excellent remarks about the didactic instrument called "the teaching unit" wants to convince the academic profession that this didactic tool is worthy of the research attention of academicians. However, he notes a problem with "the teaching unit" in that it is the standard tool of good teachers, who till now were the only ones paying attention to it. Therefore "since the design of teaching has been considered as a mediocre task normally done by teachers and textbook authors, why should anyone" – he asks – "anxious for academic respectability stoop to designing teaching and put himself or herself on one level with teachers?"

Having thus defined the extent of the social gap separating the profession and demonstrating how that gap hampers the investigative interest of academic educators, he has to now extricate the teaching unit from its grip. The author continues – " That teachers take part in design can be no excuse for mathematics educators to refrain from this task. On the contrary: the design of substantial teaching units…is a most difficult task that must be carried out by the experts in the field. By no means it can be left to the teachers, although teachers can certainly make important contribution within the framework design provided by the experts."

What we see in this argument is the process of disappropriation of the teaching profession from its professional tool. Such a disappropriation decreases the communication and increases the trust gap between both communities. Teaching-Research offers a different solution to the problem suggesting the development and strengthening of teachers' research knowledge and skills so that together with their intuitive knowledge as the guide, they can investigate fully all the beneficial consequences of the teaching unit. By increasing teachers' research capability, the gap is made narrower.

Two Approaches to Integration between Teaching Practice and Research

For an educational researcher, according to [Brown, 1992], the prospect of leaving the artificial environment of the psychological laboratory and with the help of the Design Experiment to enter the rich, complex, and constantly changing environment of the classroom is an unusual and exhilarating opportunity to observe and to understand the psychology of pedagogical dynamics directly in vivo. To engineer innovative educational environments and simultaneously conduct experimental studies on them is the content of the Design Experiment methodology.

On the other hand, the NCTM-2000 Standards asserts that the teaching of mathematics is a complex practice in that it "must balance purposeful, planned classroom lessons with the ongoing decision-making that inevitably occurs as teachers and students encounter unanticipated discoveries or difficulties that lead them to uncharted territory" (p.18). To navigate this terrain and manage its inherent dilemmas [Lampert 2001], the ideal standpoint is that of a teacher-researcher, a professional who investigates the teaching/learning processes in her/his own classroom and uses this knowledge for furthering instruction and for the navigation in the 'uncharted territories'.

Hence, for a researcher, the complexity of the classroom appears as the multitude of variables and their mutual relationships, whereas for the teacher, the same complexity appears in the moment of decision when he/she has to face the unknown during the classroom period, and react to it in a well chosen time and through well chosen comments, which utilize the unknown to better student understanding. It becomes clear that the integration of research with classroom practice is a very subtle process, which involves the composition of two, distinctly different ways of thinking about the classroom reality, two sets of different goals, and consequently, two different ways of asking questions. Both ways of thinking are necessary in the contemporary classrooms seen as a laboratory.

Similarities and Differences

There are two main methodological similarities between the approaches:

-Both of them rely on the cyclical nature of classroom research, which allows repeatedly refining the instruction and deepening the theoretical understanding of the issue at hand. Significantly, the researchers generally start their design experiments with the theoretical point of view to be verified or assessed through the classroom teaching experiment [The Design-Based Collective, 2003; Asiala, 1996], while the Teacher-Researcher usually starts with the practice of instruction [Malara, 2002], its observation and redesign followed by the possible formulation of the theory. Yet both approaches require full cycles of instruction/data collection/theoretical analysis and redesign of the intervention.

-Both approaches agree also that the "central goals of designing learning environments and developing theories…of learning are intertwined" (The Design Based Collective, 2003, p.5).

The methodologies differ significantly in the set of priorities. Teaching-Research is primarily concerned with the improvement of instruction in the classroom, and it draws its theoretical hypotheses out of particular instances of that improvement. The Design Experiment emphasizes the creation and development of theories of learning as its primary goal, with the improvement of learning process in a particular classroom seen as

the secondary goal [Cobb et al, 2003]. Similarly the role of the teacher is significantly different in both methodologies. In Teaching-Research methodology, the teacher is the main investigative agent and he/she organizes the classroom activities to fit them into the established research questions [Jaworski, 1994]; the Design Experiment has the teacher as, at most, a member of the research team, rarely as its central methodological figure whose craft knowledge serves as the main spring of investigations. In fact, [Bishop, 1998] asserts that "the research agendas …[of collaborative projects] are still dominated by the researchers' questions and orientations and not that of practitioners."
On the other hand, whether a proper response to that domination and separation is the call for the research being "closer" to practice, or being "dominated" by the issue of practice as the same author [Bishop, 1998] and [Malara and Zan, 2002] suggest, is equally questionable. What's needed, it seems, is the steady process of building the connections in two directions, new relationships between the two levels of knowledge, and consequently, between the two communities. That is the aim of Teaching-Research (NYCity model) defined below.

Teaching-Research (TR-NYC model): Examples of Application

A new model of bi-directional Teaching-Research called TR-NYC model has been formulated recently in community colleges of CUNY [Czarnocha, 2002], the goal of which is to integrate fully, educational research with teaching practice. The new model proposes an increase of validity and trustworthiness of classroom research through its coordination with modern cognitive and constructivist theories of learning. The process of coordination can proceed along two routes. A teacher-researcher can use a general theory of learning at the very beginning of the teaching-research process in the classroom, organize its practice in accordance with the suggestion of the theory or research and investigate its effectiveness using theory - based criteria [Baker, Czarnocha, 2002], or the teacher-researcher can perform a classroom teaching experiment designed on the basis of practical craft knowledge and then seek an appropriate general theory to explicate the empirical results [Czarnocha and Prabhu 2002]. In other words, the Teacher-Researcher working in accordance with these principles is fully capable to traverse the Zone of Proximal Development between the craft knowledge of the teachers and research knowledge of researchers, in both directions: from educational theory to teaching practice and from teaching practice to educational theory.

Example 1: From practice to theory [Czarnocha and Prabhu, 2000]
Observations of student learning in mathematics remedial classroom at the bilingual (Spanish/English) community college suggested that increased use of natural language in the mathematics classroom, may influence the acquisition of English as a Second Language in the domain far removed from mathematics itself. The ESL methodology of paired courses suggested itself as the organizational framework for a teaching experiment, and a pair of linked courses Intermediate ESL and elementary algebra had been formed. They were linked in two aspects: the same students attended them and the syllabi were integrated to the maximum level possible without endangering the integrity of each of the subjects. The mathematics instructor verbalized instruction to a maximal

degree possible, while the ESL instruction incorporated mathematical terminology and syntactical structure of the mathematical discourse into the English course. The results of the experiment were very interesting and encouraging: the passing rate in ESL course increased by 14 per cent (three students more passed as compared with similar courses of the same instructor. Student final essays on the topic :In between Two Cultures, where students---Dominican immigrants to NYC---were describing their impressions of immigration. The ESL instructor judged these essays as more cohesive and coherent as compared to similar essays in non-experimental classroom. The increase of the cohesiveness was due to the increased number of connectors and subordinated clauses in the essays. The hypothesis was fully confirmed: the general increase of connectors was around 15 per cent as well, but the mechanism of the process was a mystery. How mathematics discourse in algebra could have increased the number of connectors in natural language, was the main question. Here is the crucial methodological step: The coordination of the classroom situation with Vygotsky theory suggested a possibility that highly systematized thinking schema of algebra was influencing the emerging schema of the language (ESL), enriching its relational structure, increase of connectors and subordinating clauses. The concept of relative Zone of Proximal Development between algebra and language seemed to be useful in explaining this interesting process.

Example 2 From theory to practice

Research literature has amply documented calculus students' misconception about the limit of a convergent sequence [Davis and Vinner,1986]. These difficulties arise from a mental conception of the sequence and its limit through a never ending, step-by-step dynamic approach. The language used by textbooks defining horizontal asymptotes and the limit in a similar manner, adds to student confusion. Hence, students in calculus classes have intuitive (spontaneous) as well as cultivated difficulties with the concept of the limit of a convergent sequence. This topic is at the basis of fundamental calculus themes such as the definite integral and hence addressing it is of crucial importance. In preliminary studies, the source of the misconception/inability to form a coherent schema was determined to be the absence of a shift of attention from the dynamic term-by-term approach to the limit of the sequence. In order to facilitate this shift of attention, we employed the concrete context of the race between Achilles and the Tortoise from the classical Zeno paradox. Students wrote four essays at different times of the semester, each of which was assigned after certain specific items such as the geometric definition of convergence, limit laws, Weierstrassian definition was discussed in class. Questions on each essay assignment asked students to focus their attention from the perspective of the limit and its neighborhood. The first essay was assigned on the first day of class and this essay assessed students' spontaneous understanding of convergence in this context. The essays were graded with individual feedback. Explicit comments were not provided; however, each student proceeded based on instructor comments to refine their understanding in the following assignment. Students were free to use the definition of their choice, however, it was required that they demonstrate how their reasoning constituted valid justification. The entire concept acquired visual meaning from the geometric definition, computational meaning from the limit laws and analytic meaning from the Weierstrassian definition. Below are the excerpts of one student over the course

of the four essays, whose detailed developmental analysis vis-à-vis the limit is not included due to space limitations:

Quote 1:

Achilles a1-------a2-------a3-------a4-----a5------a6

5miles 5miles 5miles 5miles 5miles

Tortoise h1-------h2-------h3-----h4------h5

Let us imagine that the race is 25 miles long with a check point at every 5 miles, so that a1→ a2 = 5 miles., h1→ h2 = 5 miles, and so on. Let's also imagine Achilles runs 10 mph and the Tortoise runs 5 mph, since Achilles is faster. Knowing this, we can determine that it will take Achilles 30 minutes to run between each point…

Therefore, 30 minutes into the race, Achilles would be at point a2 and the Tortoise would be between h1 and point h2. An hour into the race, Achilles would be at point a3 and the Tortoise would be at point h2. Note a3 = h2. An hour and a half into the race, Achilles would have passed the Tortoise and be at point a4, while the Tortoise is between the point h2 and the point h3.

Though Zeno's conclusion that the quicker will never pass the slower (if given a head start) may have been valid during the fifth century B.C., it does not stand true to today's experiences.

In the first essay that determines students' spontaneous understanding this student has determined that the race will be won by Achilles.

Quote 2:

As you can see, Zeno's conditions would never be satisfied because it is impossible for the Tortoise to win the race. Achilles would always win because for every 10 seconds, he increased his position by 100 m. On the other hand, the Tortoise only increases his position by 10m every 10 seconds.

Here, the students' reasoning is not correct in spite of the correct conclusion, since the conditions of the paradox do not imply that measurements are made at equal time intervals.

Quote 3:

The sequence of distances is an increasing sequence and is bounded above since once Achilles passes the Tortoise the race is over. The smallest number greater than every term in the sequence is 11.2 s. The limit of the sequence is the point at which Achilles and the Tortoise intersect. The slope of the line for Achilles is y = 10x and the slope for tortoise is y = x+100. To figure out the limit

$10x = x + 100$

$9x = 100$

$x = 100/9$

$x = 11.11$

The proof implies Achilles passes the Tortoise.

Note that in spite of his use of the word proof, he has not used the geometric or Weierstrassian or Least Upper Bound Axiom (LUB) for the proof. He begins the use of the LUB axiom, however, does not continue that line of reasoning and reverts to the intersection of the two graphs.

Quote 4:

However, the sequence in this particular problem is {…, 111.1, 111.11, 111.111, …}; which can also be written as {…, 111.$\underline{1}$}. In situations like this we can implicitly use infinite sums.

111 1/10 + 111 1/100 + 111 1/1000 + 111 1/10000 + … = 111

Observe that as you add more and more terms, the partial sums become closer and closer and closer to 111 1/9. Therefore, we can say 111 1/9 is the smallest number greater than every term of the sequence (and also the limit of the sequence).

To prove that 111 1/9 is the limit of the sequence, we can apply the geometric definition of convergence of a sequence.

In the last and final essay he is able to provide a proof using the material covered in class and his use of the geometric definition in the text and the drawings illustrate the successful coordination of the A & T generated sequence with the definition of the limit based on the LUB axiom.

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Notes

[1] Described movement between the two levels of knowledge reminds of the movement within the Zone of Proximal Development between the "spontaneous" conceptions of teachers and "scientific" conceptions of researchers. Integration of the two may be taking place within so defined ZPD.