Premature Mortality and Poverty Measurement

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Abstract

There is a glaring paradox in all commonly used measures of poverty. The death of a poor person, because of poverty, reduces poverty according to these measures. This surely violates our basic intuitions of how poverty measures should behave. It cannot be right in concept that differentially higher mortality among the poor serves to reduce poverty. This paper begins the task of developing poverty measures that are not perversely mortality sensitive. A family of measures is proposed that is an intuitive modification of standard poverty measures to take into account the fact that the rich live longer than the poor.

Key words: *Premature Mortality, Life Time Income Profile, Poverty Measure, Characterization, Steady State Population*

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1 Introduction

More than a quarter of a century ago, **Amartya Sen (1976)** pointed out a glaring paradox in the most commonly used measure of poverty - the head-count ratio. He observed that taking income away from the poor did not change this measure, since it did not change the number of people in poverty. This, he argued, violated our basic intuitions about poverty. This simple but powerful observation led to the development of "distribution sensitive" poverty measures, including the Sen measure (**Sen, 1976**) and the FGT measure (**Foster, Greer and Thorbecke, 1984**), which have become the workhorse poverty measures in applied and policy work.

But there is perhaps an even more glaring paradox in the head-count ratio and it is this. If a poor person dies, poverty decreases. This also holds true for the distribution sensitive measures of poverty such as the commonly used FGT family of measures. Reduction of poverty through deaths of the poor, due to poverty, must surely violate our basic intuitions on poverty. It cannot be right in concept that differentially higher mortality rates among the poor serve to reduce poverty. This conceptual challenge is only strengthened by the fact that higher mortality rates and lower life expectancies among the poor, because of their straitened condition, are an established empirical regularity the world over. Similar issues are also discussed in the area of health economics. An untimely demise of the poor, who are usually not in a very good state of health, has the perverse implication that the average health standards of the society improves.

Of course, the paradoxes of population variation and welfare measurement have been discussed by philosophers and economists over many years. **Parfit's (1984)** "Repugnant Conclusion" launched the modern debate. As formulated by **Arrehnius (2000)**, this is a critique of Total Utilitarianism since: "For any perfectly equal population with very high positive welfare, there is a population with a very low positive welfare which is better." What is repugnant is that one society can be pronounced to be better than another even though every person in the former is worse off than every person in the latter, simply because population in the first is so much higher than in the second. Issues of length of life and standard of living during life are also discussed in the economics literature, for example in **Blackorby, Bossert and Donaldson (1995, 99)** and **Karcher, Moyes and Trannoy (1995)** among others. Focusing on the average utility as a way of getting around the repugnant conclusion has its own problems because, as **Cowen (1989)** rightly observes, “average utilitarianism cannot escape recommending the death of all those below the social mean.” In the case of standard poverty measures, this paradox manifest itself through the conclusion that killing people below the poverty line will reduce poverty.

The standard approaches to poverty measurement look at a snapshot of alive individuals. Hence the paradox that when a poor individual disappears, measured poverty goes down. The obvious answer to this paradox is to not let individuals disappear because of death, to keep them nevertheless in the universe of individuals whose poverty is being measured. We call the set of individuals who enter into the measurement of poverty, whether they are alive or dead, the *relevant set* of individuals. It should be clear that the relevant set of individuals is a deeply normative concept. Who should we consider? All those individuals who have ever lived? Should deaths due to poverty in the middle ages burden poverty measurement today? We believe that a good starting point is to specify a normative lifetime L, close to the top of the range observed in rich countries today. In other words, the relevant set is all individuals born L years ago or later.

Mortality brings to the fore the question of missing individuals discussed above. But attempting to address this question by defining a relevant set of individuals raises another important issue. We started off by attempting to define poverty today, but find ourselves considering individuals whose incomes are dated from earlier periods. This brings the time factor centrally into consideration. We can no longer restrict attention to income at a point in time, but have to take into account "time profiles of income" - of those who died recently but also of those who are currently alive. The problem can then be specified in several steps. First, develop a general way of measuring poverty given the time profiles of income for an arbitrary set of individuals. Second, define the relevant set of individuals relative to a normative life span. Third, combine the two methods to measure poverty when there is premature mortality. These are the steps followed in this paper.

This paper draws its inspiration from the large literature on population and welfare measurement, but its focus is on the measurement of poverty. Its objective is to launch a discussion that will, hopefully, lead to the development of poverty measures that are not vulnerable to the mortality paradox. Section 2 starts with a development of poverty measurement in dynamic settings. Basic notation and definitions are introduced. We demonstrate a general theorem regarding the form of poverty measures in this section. Formal definitions and proofs are relegated to an appendix. Section 3 introduces premature mortality and discusses the modifications to be incorporated in poverty measurement to take care of this. Section 4 goes on to discuss several examples of poverty measures that can be used in empirical applications. This section also axiomatizes lifetime poverty measures based on intertemporal considerations. Finally, Section 5 offers some concluding remarks.

2 Poverty and Time: General Results

Let us start with some notation. We will denote the set of real numbers by R and the non-negative reals by R_+. The set of integers is given by Z and the set of positive integers by Z_+. Let $H = \bigcup_{n \in Z_+} Z^n$, Z^n being the n-dimensional cartesian product of Z. We will be considering discrete periods of time in this paper. Denote $\bigcup_{n \in Z_+} R^n_+$ by Ω, the set of all possible income distributions, R^n_+ being the n-dimensional cartesian product of R_+.

Let there be n individuals. Each person i is completely characterized by her birth date $t_i \in Z$, actual length of life $l_i \in Z_+$ and the life time income profile $Y_i = (y_{i,1}, ..., y_{i,l_i}) \in R^{l_i}_+$ for $i = 1, .., n$, where a typical element $y_{i,l}$ is person i's income in period l of her life. To construct the income profile for those who are aged $< l_i$ now (and hence the income levels for the remaining periods of their lifetime are unknown) we will need to use some simplifying assumptions on fitting an income trend function $T(.)$ based on the observable income levels (upto the current period). We assume that (i) T is increasing in each argument, (ii) $T(x, ..., x) = x$ for $x > 0$. If person i is currently aged k_i ($< l_i$) then his lifetime income profile (with l_i elements) is estimated as $(y_{i,1}, ..., y_{i,k_i}, \tau_i, ..., \tau_i)$ where $\tau_i = T(y_{i,1}, ..., y_{i,k_i}) : \Omega \to R_+$ in general. But to ensure that a person's trend value is above poverty line if he is non-poor in all periods lived so far, we additionally assume that (iii) if $y_{i,l} \geq z_{t_i+l}$ for all l, then $\tau_i = max\{z_{t_i+k_i+1}, ..., z_{t_i+l_i}, T(.)\}$. We need to impose certain structure on the function $T(.)$ for it to be consistent with SI (or TI). Thus the fourth assumption on T is $T(lx_1, ..., lx_m) = lT(x_1, ..., x_m)$ for any $m \in Z_+$. That is, $T(.)$ is homogenous of degree one (or $T(aU + X) = a + T(X)$ for any $X \in \Omega$, U being a vector of ones of the same order as X and any $a \in R$ such that $aU + X \in \Omega$). The set $A = \{Y_i, i = 1, ..., n\}$ is the income profile for the relevant population. Denote the vector of birth dates by $T = \{t_1, ..., t_n\}$ where each $t_i \in Z$. Thus $T \in H$. Note that for the set of relevant individuals, which we call the relevant set, $-L \leq t_i < 0$.

To discuss measurement of poverty we need to talk about a measure of subsistence requirement for the population under consideration, or the 'traditional poverty line'. Here, we need to have a subsistence requirement for each period of time. Hence, we define the vector $S = \{..., z_{-1}, z_0, z_1, ...\} \in R_+^\infty$, where the current period is period 0 and $z_t \in R_+$ represents the poverty line for period t. R_+^∞ is the set of positive real valued vectors of infinite length.

Let us now define our life time measure of poverty in the most general form as a function

$$P = P(A, T, S, n) : \Omega \times H \times R_+^\infty \times Z_+ \rightarrow R_+. \tag{1}$$

So, we assume that the measure of poverty is a positive real value and it depends on the life time income profile for the population, the birth dates of each of the members of the population, the subsistence requirement vector and the size of the population. For person i, the k^{th} period of life is considered to be spent in poverty if $y_{i,k} < z_{t_i+k}$, that is, her income for age k fell below the subsistence requirement for the period when she was aged k. Note that, now we can not talk about a 'person' being poor; rather, all we can now say is that such and such periods of her life have been spent in poverty. The censored income profile associated with Y_i is denoted by Y_i*, whose typical element is $y_{i,k}* = min\{y_{i,k}, z_{t_i+k}\}$. Note that if t_i = -1 and l_i = 1 for all i, that is everybody is born at date "-1" and lives for 1 period, then (1) reduces to the standard snapshot poverty measure.

The poverty index is supposed to satisfy certain desirable properties. We describe them as follows. (As noted in the introduction, the formal statements of these are collected in **Appendix 1**.)

Continuity (C): A small change in income should not cause jumps in the poverty measure and minor observational errors in incomes will generate minor changes in the poverty index.

Focus (F): The poverty index should be independent of the incomes in excess of the subsistence requirement. So, person i's income for period l will not affect the poverty measure if she was nonpoor in period l.

Monotonicity (M): A reduction in the income of a person in any period when she was poor must increase poverty.

Symmetry (S): Any characteristic other than the life time income profile and birth date, e.g. the names of the individuals, is irrelevant to the measurement of poverty.

Scale Invariance (SI): The poverty index must be independent of the unit of measurement for income and subsistence requirement in any period.

Translation Invariance (TI): For any period, an equal increment in income for all persons and the subsistence requirement do not affect the poverty measure.

Population Principle (P): This is the replication invariance principle that implies that if we make a m-fold copy of the population, all other things unchanged, then the poverty index is unaffected.

Interpersonal Transfers Principle (TR): A transfer of income in any period, to person i who is poor in that period from another person j who is also poor in the same period but richer than i, without changing their relative position for that period, will reduce poverty.

Subgroup Decomposability (D): This is a requirement similar to the subgroup monotonicity property of FGT and also similar to (but stronger than) the subgroup consistency axiom of **Foster and Shorrocks (1991)** which requires overall poverty for a population partitioned into subgroups to increase if poverty in one or more subgroups increases and stay constant in others.

We will now state the first and basic result of this paper regarding some benchmark features of the poverty measure given by (1) when it is required to satisfy some of the above mentioned properties.

Theorem 1: The poverty measure in (1) satisfies C, F, M, S, P, TR and D if and only if, for all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n) \in \Omega$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} \phi(Y_i*; z_{t_i+1}, ..., z_{t_i+l_i}) \tag{2}$$

where $\phi : \Omega \times \Omega \to R_+$ is continuous, decreasing and strictly convex in each of the arguments $y_{i,k}*$, $k = 1, ..., l_i$ of Y_i*.

Theorem 1 identifies a general class of measures that are useful for application to data and is simply parametrized. We will now look at the consequence of invoking alternative invariance assumptions on the poverty measure in (2). Here one can establish the following results.

Theorem 2: (a) The poverty measure given by (1) satisfies C, F, M, S, P, TR, D and SI if and only if for all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for life time income matrices $A = (Y_1, Y_2, ..., Y_n)$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}\phi_R(R_i) \tag{3}$$

where $R_i = (r_{i,1}, r_{i,2}, ..., r_{i,l_i})$ with $r_{i,k} = \frac{y_{i,k}*}{z_{t_i+k}}$, for $k = 1, ..., l_i$. $\phi_R : \Omega \to R_+$ is continuous, decreasing and strictly convex in each of the arguments $r_{i,k}$, $k = 1, ..., l_i$ of R_i.

(b) The poverty measure given by (1) satisfies C, F, M, S, P, TR, D and TI if and only if for all $n \in Z_+$, $S \in R_+^{\infty}$, $T \in H$ and for life time income matrices $A = (Y_1, Y_2, ..., Y_n)$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}\phi_A(A_i) \tag{4}$$

where $A_i = (a_{i,1}, a_{i,2}, ..., a_{i,l_i})$ with $a_{i,k} = z_{t_i+k} - y_{i,k}*$, for $k = 1, ..., l_i$. $\phi_A : \Omega \to R_+$ is continuous, increasing and strictly convex in each of the arguments $a_{i,k}$, $k = 1, ..., l_i$ of A_i.

3 Premature Mortality, the Extended Profile and Poverty Measurement

We will now try to tackle the issue of premature mortality, as indicated in section 1. That is, we will now ask how the length of life interacts with the income level and if so, how should we incorporate that into the poverty measure discussed above.

Consider two populations of n persons all born at the same date. In each, there are 2 poor persons with income y_1 and y_2 ($> y_1$) in each period of their lives and (n-2) persons who are non-poor throughout. All persons in population 1, all non-poor and the richer poor in population 2 lives for two periods. The poorest in population 2, because of their impoverished condition, live only for one period.

Now, let us compare traditional snapshot poverty levels in the two periods. In the first period, the income distributions are identical, so all usual measures will show the same level of poverty for both the populations. In the second period, poverty in population 2 will be lower according to any standard poverty measure *because the poorest person has died*! In fact, even if we use the life time poverty measure developed in the last section based on the life time income profiles of the persons in these two populations, poverty would still be unambiguously no higher in population 2 than in 1. Something must be wrong.

To solve the above anomaly, we proceed in the following manner. For any population, let us consider a normative length of life (say L); the length of time each person in the population is expected to live upto. Choice of the normative length of life, L, is a crucial issue for this life time poverty measurement. The poverty ranking of a population may change with respect to other population groups if the value of L is changed, say from 80 to 70 years. Thus, a value judgement on L is an important element of this analysis.

If an individual dies before the age L ($l_i < L$) then one has to take note of this premature death in the life time income profile itself. To facilitate this, one has to extend (truncate) the income profile of person i from Y_i of length l_i to, say, $\widehat{Y_i}$ of length L. To achieve that, if $l_i < L$ then for person i, the income values are taken to be $E(y_{i,1}, ..., y_{i,l_i})$, $E : \Omega \rightarrow R_+$, in each of the periods $l_i + 1, ..., L$. Thus, if person i dies at age $3 < L$, then for this person we have the income profile $\widehat{Y}_i = (y_{i,1}, y_{i,2}, y_{i,3}, e, ..., e) \in R_+^L$ where $e = E(y_{i,1}, y_{i,2}, y_{i,3})$, this function E gives us a proxy for the fictitious "income" of a dead individual. We assume that (i) E is increasing in each argument, (ii) $0 < E(x, .., x) \leq x$ for $x > 0$ and (iii) $E(x_1, .., x_m) \geq z_j$ for $j = m+1, ..., L$ if $x_i \geq z_{t+i}$ for all i and for any $m \in Z_+$ (t being the birthdate of this person). These assumptions ensure that the proxy income is positive and less than the living income. That is, a person is better off alive. Thus, implicitly we are imposing a welfare condition that values longevity. One could argue in support by saying that the society values each individual positively in terms of their potential to increase social welfare.[1]. Thus the untimely absence of any individual should reduce social welfare and analogously, if the person happened to be poor, this should increase social poverty[2].

Also, if a person was rich in all periods of his lifetime, then the proxy income should also be above the poverty line. To ensure this, we redefine the hypothetical income for a person who was non-poor in all periods of her life as (iii) $e_i = max\{z_{t_i+k_i+1}, ..., z_{t_i+L}, E(.)\}$ if $y_{i,l}.z_{t_i+l}$ for all $l = 1, ..., l_i$. This assumption

[1] A similar interpretation in terms of reducing the burden of national debt is found in **Duclos and Makdissi (2004)**.

[2] One can also think of a situation where the death of a poor person may be considered as putting him out of his misery and hence improves the fictitious individual's well being. But this would immediately bring us back to the discussion involving the repugnant conclusion.

implies that a dead rich person's presence will not affect any poverty measure that focusses only on the incomes of the poor, except through the size of the relevant population. As for the function $T(.)$, we can impose analogous restrictions on $E(.)$ for it to be consistent with SI (or TI). When $l_i > L$, we ignore the income of this individual for periods $L+1, ..., l_i$. As L should be chosen to be suitably high, loss of information due to this truncation is expected to be negligible. Call the extended (truncated) income profile for the population $\widehat{A} = \{\widehat{Y}_i, i = 1, ..., n\}$.[3]

To illustrate the construction of lifetime and extended lifetime income profiles, consider a population with two individuals, $L = 5$, $l_1 = 4$ and $l_2 = 5$. Both are aged 3 today. So $Y_1 = (y_{1,1}, y_{1,2}, y_{1,3}, \tau_1)$ where $\tau_1 = T(y_{1,1}, y_{1,2}, y_{1,3})$ and $\widehat{Y_1} = (Y_1, e)$ where $e = E(Y_1)$. $Y_2 = \widehat{Y_2} = (y_{2,1}, y_{2,2}, y_{2,3}, \tau_2, \tau_2)$ where $\tau_2 = T(y_{2,1}, y_{2,2}, y_{2,3})$.

We now focus our attention on the possible alternative specifications of the function $E(.)$. Lets take a look back at our example of two populations discussed at the beginning of this section. If we assume that $E(y_1) = y_1$ as the second period proxy income for the poorest individual in population 2, then the poverty profiles will be identical for both the populations using the modified normative life time measures. That is, the poverty measure becomes insensitive to the fact that the poorest died early in population 2. Thus, the substitution of $E(y_1) = y_1$ (as done here) achieves a neutrality to death for our life time poverty measure, whereas earlier (without the correction for premature mortality) it was perversely affected by mortality (poverty reduces if poorest people die off). If we take $E(y_1) < y_1$, then this is consistent with the "better off alive" assumption. This way a penalty can be imposed on premature mortality. An alternative and equally exciting approach to this issue is through formulating a function that links the survival probability to the number of spells of poverty. That would result in considering the direction of causality from poverty to premature death. We are not attempting that in the present paper but only considering the joint impact here.

So, we define the aggregate normative life time poverty measure defined on the extended profile $\widehat{A}$ for the population under consideration by

$$P^e = P^e(\widehat{A}, T, S, n) : \Omega \times H \times R_+^\infty \times Z_+ \to R_+. \tag{5}$$

One can now invoke the assumptions laid out in section 2 on the extended income

[3]This construction is similar in spirit to the *critical-level value function* in **Blackorby and Donaldson (1984)** for evaluating population changes.

profiles $(\widehat{Y}_1, \widehat{Y}_2, ..., \widehat{Y}_n)$, in an analogous fashion to theorem 1 and 2, to arrive at aggregate life time poverty measures.

Theorem 3: (a) The normative life time poverty measure given by (5) satisfies C, F, M, S, P, TR, D and SI if and only if for all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for normative life time extended income matrices $\widehat{A} = (\widehat{Y_1}, \widehat{Y_2}, ..., \widehat{Y_n})$, with $\widehat{Y_i} = (\widehat{y_{i,1}}, ..., \widehat{y_{i,L}})$, say, for $i = 1, ..., n$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}\phi_R^e(\widehat{R_i}) \tag{6}$$

where $\widehat{R_i} = (\widehat{r_{i,1}}, ..., \widehat{r_{i,L}})$, $\widehat{r_{i,k}} = \frac{\widehat{y_{i,k}*}}{z_{t_i+k}}$ for $k = 1,..., L$. $\phi_R^e : \Omega \to R_+$ is continuous, decreasing and strictly convex in each of the arguments $\widehat{r_{i,k}}$, $k = 1,..., L$ of $\widehat{R_i}$.

(b) The normative life time poverty measure given by (5) satisfies C, F, M, S, P, TR, D and TI if and only if for all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for normative life time extended income matrices $\widehat{A} = (\widehat{Y_1}, \widehat{Y_2}, ..., \widehat{Y_n})$, with $\widehat{Y_i} = (\widehat{y_{i,1}}, ..., \widehat{y_{i,L}})$, say, for $i = 1, ..., n$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n}\phi_A^e(\widehat{A_i}) \tag{7}$$

where $\widehat{A_i} = (\widehat{a_{i,1}}, ..., \widehat{a_{i,L}})$, $\widehat{a_{i,k}} = z_{t_i+k} - \widehat{y_{i,k}}*$ for $k = 1,..., L$. $\phi_A^e : \Omega \to R_+$ is continuous, increasing and strictly convex in each of the arguments $\widehat{a_{i,k}}$, $k = 1,..., L$ of $\widehat{A_i}$.

4 Examples and Special Cases

4.1 Examples

The above theorem characterises a general class of aggregate life time poverty measures that satisfy the set of axioms put forth. The final choice for a practitioner may be any particular member of this class and specific forms will follow from specific assumptions that are taken on the form of the functions ϕ_R^e and ϕ_A^e in theorem 3. Below we provide a few examples.

Simple illustrations of measures belonging to the class characterised by theorem 3 (a) is given by the following two examples.

Example 1: Consider

$$\phi_R^e(\widehat{R_i}) = \frac{1}{L}\sum_{k=1}^{L}(1 - (\widehat{r_{i,k}})^{\delta}),$$

for all $i = 1, ..., n$. So that the extended life time poverty measure becomes

$$P^e(\widehat{A}, T, S, n; \phi_R^e) = 1 - \frac{1}{n}\sum_{i=1}^{n}\frac{1}{L}\sum_{k=1}^{L}(\frac{\widehat{y_{i,k}*}}{z_{t_i+k}})^{\delta}, \tag{6.1}$$

where $0 < \delta < 1$ is some constant. For $\delta = 1$, this is analogous to average of income gap ratio.

Example 2: An analogue of the FGT measure arises when we consider

$$\phi_{R,FGT}^e(\widehat{R_i}) = \frac{1}{L}\sum_{k=1}^{L}(1 - \widehat{r_{i,k}})^{\alpha}$$

for all $i = 1, ..., n$ again. Then $P^e(.)$ becomes

$$P^e(\widehat{A}, T, S, n; \phi_{R,FGT}^e) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{L}\sum_{k=1}^{L}(\frac{z_{t_i+k} - \widehat{y_{i,k}*}}{z_{t_i+k}})^{\alpha}, \tag{6.2}$$

where $\alpha > 1$ is some constant.

Again, a simple example of measures belonging to the class characterised by theorem 3 (b) is given in example 3 below.

Example 3: Consider

$$\phi_A^e(\widehat{A_i}) = \frac{1}{L}\sum_{k=1}^{L}(\widehat{a_{i,k}})^{\alpha}$$

for all $i = 1, ..., n$. Then,

$$P^e(\widehat{A}, T, S, n; \phi_A^e) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{L}\sum_{k=1}^{L}(z_{t_i+k} - \widehat{y_{i,k}*})^{\alpha}, \tag{7.1}$$

for some constant $\alpha > 1$. For $\alpha = 1$, this can be written as

$$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{L}\sum_{k=1}^{L}(z_{t_i+k} - \widehat{y_{i,k}*}) = \frac{1}{n}\sum_{i=1}^{n}(\frac{1}{L}\sum_{k=t_i+1}^{t_i+L} z_k - \widehat{\bar{\bar{Y_i}}*}), \tag{7.2}$$

the average shortfall over all the individuals' life times.

Note that these classes of measures closely resemble the aggregate deprivation measures discussed in **Mukherjee (2001)**. As poverty can be seen as a measure of deprivation arising due to shortfall from a subsistence level, this proximity is quite natural.

Example 4: Consider the following functional form that is discussed in **Tsui (2002)**.

$$\phi_R^e(\widehat{R_i}) = \Pi_{t=1}^L \widehat{r_{i,t}}^{-\alpha_t} - 1$$

for $\alpha_t \geq 0$ for all t and all i. Then the poverty measure becomes

$$P^e(\widehat{A}, T, S, n) = \frac{1}{n}\sum_{i=1}^{n}(\Pi_{t=1}^L \widehat{r_{i,t}}^{-\alpha_t} - 1) = \frac{1}{n}\sum_{i=1}^{n}\Pi_{t=1}^L \widehat{r_{i,t}}^{-\alpha_t} - 1. \tag{6.3}$$

Example 5: As a final example, consider the functional form given by

$$\phi_R^e(\widehat{R_i}) = -\frac{1}{L}\sum_{t=1}^{L} ln\widehat{r_{i,t}}$$

for all i = 1,..., n. Then $P^e(.)$ is given by

$$P^e(\widehat{A}, T, S, n) = -\frac{1}{nL}\sum_{i=1}^{n}\sum_{t=1}^{L} ln\widehat{r_{i,t}} \tag{6.4}$$

4.2 Characterizing Intertemporal Separability

A key feature of the examples 1, 2, 3 and 5 given above is the element of intertemporal separability in characterizing individual poverty based on her life time profile. Since separability will turn out to be a characteristic of many operational measures, in this subsection we provide an axiomatic characterization of intertemporal separability before moving to a rigorous derivation of an extended FGT measure.

So far, we have not discussed any structural assumptions regarding the intertemporal properties of our life time poverty measure. How the different periods of a person's life time are compared is the issue of the next two axioms. (Again, the formal statements are collected in **Appendix 1**.) These two will be used to discuss the issue of intertemporal aggregation.

Intertemporal Symmetry (ITS): This is the symmetry requirement across time which ensures that any time period of a person's life time has the same significance with respect to life time poverty calculations. Suppose we have two otherwise identical populations with the difference that in one everyone is poor only in the first period of their life and in the other only in the last period. Then (ITS) demands that aggregate poverty will be the same for the two populations. (ITS) requires that lifetime poverty of a population does not depend on the timing of poverty spells (shifting of poverty spells to the future has no effect on the poverty measure).[4]

Intertemporal Consistency (ITC): Consider the population income profiles A and B which differ only for individual i. (ITC) is closely related to the subgroup consistency axiom of **Foster and Shorrocks (1991)**. (SC) focusses on partitioning a population into two fixed sized subgroups while (ITC) considers the situation where the life time of any individual i is divided into two nonoverlapping intervals of time before and after some threshold period l. Y_i^1 (and X_i^1) is the truncated, at l, income profile of the individual i with original income profile Y_i (and X_i). Y_i^2 (and X_i^2) is the remaining part of the profile (from period $l+1$ onwards). (ITC) requires that if the poverty experience of this individual is exacerbated in the early periods of her life and that of the later periods remain unaffected (according to the poverty calculation assuming her birth date to be $l+1$) then her actual life time poverty experience must also become worse.

That is, we are considering the whole life time of an individual as consisting of two subintervals, one for the early periods of her life and the other for the remainder. As shown below, (ITC) provides a formulation that allows one to compute the life time poverty of an individual from the poverty levels of each period of her life time. It is also closely allied to the stronger condition of decomposability which requires that individual life time poverty is a weighted average of period wise poverty levels.

Formulations (6) and (7) allows us to breakdown the poverty level of an extended income profile into a simple average of individual level lifetime poverty indicators. Of course this simplification comes at a cost. An intertemporally separable poverty measure is not equipped to distinguish between situations of chronic and transient poverty. In the former situation, the interaction between the levels of shortfall in different periods of an individual's lifetime becomes important. This is not captured by the separable measures.

We now turn to the issue of finding a reasonable formulation for this individual lifetime poverty indicator in the following theorem.

Theorem 4: **(a)** The poverty measure given by (5) satisfies C, F, M, S, P, TR,

[4] For intertemporal measurements, it is usual to introduce a time discount factor which is relevant for individual perception based at a point of time. As we are addressing a holistic measurement considered by a welfarist social planner, such discount factors are not appropriate in the present context

D, SI, ITS and ITC if and only if for all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for extended life time income matrices $\widehat{A} = (\widehat{Y_1}, \widehat{Y_2}, ..., \widehat{Y_n})$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} G^e(\frac{1}{L}\sum_{t=1}^{L}\psi_R^e(\widehat{r_{i,t}})) \tag{8}$$

where $G^e : R_+ \to R_+$ is continuous, increasing and convex. $\psi_R^e : R_+ \to R_+$ is continuous, decreasing and strictly convex.

(b) The poverty measure given by (5) satisfies C, F, M, S, P, TR, D, TI, ITS and ITC if and only if for all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for extended life time income matrices $\widehat{A} = (\widehat{Y_1}, \widehat{Y_2}, ..., \widehat{Y_n})$, it is ordinally equivalent to

$$\frac{1}{n}\sum_{i=1}^{n} G^e(\frac{1}{L}\sum_{t=1}^{L}\psi_A^e(\widehat{a_{i,t}})) \tag{9}$$

where $G^e : R_+ \to R_+$ is continuous, increasing and convex. $\psi_A^e : R_+ \to R_+$ is continuous, increasing and strictly convex.

This theorem completely characterizes the class of extended life time poverty measures that are intertemporally separable. Note that the measures cited in examples 1, 2, 3 and 5 above are members of this class. There is an implicit value judgement involved in choosing to work with such measures as they assume that the poverty experiences pertaining to different periods of a person's life time influences the life time poverty experience independently of each other. If one does not subscribe to this judgement then the intertemporally separable class is not appropriate. One can instead consider the type of measure put forth in example 4, which is *not* intertemporally separable.

4.3 The Extended FGT Measure

To further illustrate the above theorem, consider the following form of the individual poverty indicator ψ_A^e given by $\psi_A^e(\widehat{a_{i,t}}) = (\widehat{a_{i,t}})^\alpha$ and $G^e(x) = x$. The poverty measure would then be given by

$$P^e(\widehat{A}, T, S, n; \phi_A^e) = \frac{1}{nL}\sum_{i=1}^{n}\sum_{t=1}^{L}(\widehat{a_{i,t}})^\alpha. \tag{9.1}$$

Similarly if we consider $\psi_R^e(\widehat{r_{i,t}}) = (1 - \widehat{r_{i,t}})^\alpha$ and $G^e(x) = x$, we will obtain

$$P^e(\widehat{A}, T, S, n; \phi_R^e) = \frac{1}{nL}\sum_{i=1}^{n}\sum_{t=1}^{L}(1 - \widehat{r_{i,t}})^\alpha. \tag{8.1}$$

This once again is an analogue of the FGT measure extended to the life time poverty measurement situation.[5]

Our definition of the vector $\widehat{Y}_i$ implies that if a poor person dies before the normative age, then she will be considered as if she were poor during the periods subsequent to her death. As we will show, this has extremely interesting implications in the context of our measurement methodology.

To illustrate the difference of the measures we discuss with the traditional measures of poverty that do not take into account the mortality patterns of the population under consideration, consider the following example.

Example 6: Suppose the poverty measure we are using is analogous to income gap ratio (i.e. FGT with $\alpha = 1$). Consider two populations. Assume that the normative life time for both the population is 100. The subsistence requirement is \$ 2 each period.

Suppose in population 1, the percentage of poor is 20, all of whom live for 50 years and have \$ 1 income in each period of their lifetime. The proxy income, when they are dead, is taken to be \$ 0 (as a limiting value, for simplicity). Hence, the premature mortality corrected income gap ratio for this population will be given by $I_1 = 20\% \times \left(\frac{50}{100}(1-\frac{1}{2}) + \frac{50}{100}(1-0)\right) = 15\%$. Population 2 has 18% poor who live for 30 years and also earn \$ 1 each period with proxy income \$ 0. We similarly compute $I_2 = 18\% \times \left(\frac{30}{100}(1-\frac{1}{2}) + \frac{70}{100}(1-0)\right) = 15.1\%$.

In this example, the traditional poverty gap measure and its usual life time version shows a lower value for population 2 but as the life span is much shorter, our mortality corrected measure shows a higher value for population 2 than for population 1.

4.4 Steady State

The methodology outlined above can be adapted for empirical purposes under some simplifying assumptions. For the individuals in the relevant set, we need to have data on birth dates (t_i), length of life (l_i) and income profiles (Y_i) of each member i in the population. Then one has to decide on a normative length of life L for this population to decide on the relevant set and a suitable proxy income function

[5] Note that **Rodgers and Rodgers (1993)** used this measure, for $\alpha = 2$, to measure chronic poverty in the United States. Although they did not attempt a formal characterization of this measure.

to arrive at the extended profiles, $\widehat{Y_i}$, for each (possibly hypothetical) person in the relevant set. The relevant set can be determined as discussed in section 1 above. Then, using equation (8.1) or (9.1), the aggregate poverty measure for the extended profile can be computed.

Now, if we take the income of each individual to be the same at each period of his/her life (say y_i), take $e(Y_i) = Y_i$ and take the subsistence requirements to be the same also (say z), then (8.1) and (9.1) reduces to

$$\frac{1}{\sum_{i=1}^{n} \frac{L}{l_i} l_i} \sum_{i=1}^{n} \frac{L}{l_i} l_i (\frac{z - y_i *}{z})^{\alpha} \tag{8.2}$$

and

$$\frac{1}{\sum_{i=1}^{n} \frac{L}{l_i} l_i} \sum_{i=1}^{n} \frac{L}{l_i} l_i (z - y_i *)^{\alpha} \tag{9.2}$$

respectively and finally to

$$\frac{1}{n} \sum_{i=1}^{n} (\frac{z - y_i *}{z})^{\alpha} \tag{8.3}$$

and

$$\frac{1}{n} \sum_{i=1}^{n} (z - y_i *)^{\alpha} \tag{9.3}$$

Note that even though the formulation (8.3) and (9.3) looks identical to the usual snapshot poverty measures, but they are not actually equivalent as the set of individuals consists of not only those who are currently living but also those who were ever born L years ago or later. Thus, n will be generally much larger than the current population size.

It can be seen from the above that if l_i is constant across Y_i then we recover the standard FGT index. However, if income and length of life are positively correlated, the P^l measure will be higher than the traditional poverty measure P and if there is negative correlation the P^l measure will be lower than P. In any event, measured poverty will be affected by the income lifetime relation, over and above the distribution of income. The measure given above is implementable given the wealth of information on the relationship between income and expected length of life that is available nowadays for the developed countries and even for some of the developing ones.

5 Conclusion

In the presence of premature mortality for the poorer sections of the population, standard snapshot poverty measures will show a decrease. To avoid this welfare measurement paradox, in this paper we develop and characterise a poverty measure based on the life time income profile of an individual.[6] This measure does not exhibit such paradoxical behaviour but one can further modify this measure, defining it on a normative rather than actual life time of the individuals so that premature mortality of the poor actually effects the poverty measure positively. We characterise and illustrate such a measure here and indicate how to compute this measure in practice in a simple fashion. Choice of L, as mentioned earlier, is a crucial issue for this life time poverty measurement. A value judgement on L is an important element of this analysis. Also, ascribing income to the very young or the very old may be a problem but this can be handled by using the per capita equivalent income in the household.

We could have put forth a two-dimensional snapshot measure of well-being as a solution to this problem, with poverty and life expectancy of the population as the two determinants. But that would still have missed looking at the poor who are actually, but albeit prematurely, dead. Here, our focus was specifically on the inter-dependence of poverty and mortality. We know that the poor have higher natality but we do not incorporate this in our measure explicitly. As we are considering all the persons who were born L years ago or later, the natality factor is automatically taken care of. Also, premature mortality due to some nonincome related factors (such as accidents) affect rich and poor alike and such causes would net out in the aggregate. So, we do not consider such factors here. But if the rich were to have a lower life expectancy because they eat too much or prefer dangerous sports, then this would get reflected in the poverty measure and make it biased (lower poverty). This is a caveat that needs to be taken care of in applied work.

This measure takes account of vital events like birth and death. But one could think of other important demographic events like immigration and emigration that affects the poverty status of a population. These issues are beyond the scope of our structure, but one might put forward a tentative solution as follows. An immigration or emigration results in a left or right truncated income profile for the relevant indi-

[6]The problem of using only current income for inequality measurement has been noted in the literature. For example, **Millimet, Podder, Slottje and Zandvakili (2003)** put forth an approach to bound life time income inequality using only cross sectional data.

vidual. For example, a profile like $(y_1, ..., y_k)$ or $(y_{k+1}, ..., y_l)$ where l is the life time of the individual and $k < l$ is the year of transition. One may now complete these vectors as $(y_1, ..., y_k, z_{k+1}, ..., z_l)$ or $(z_1, ..., z_k, y_{k+1}, ..., y_l)$, where z_t is the subsistence requirement relevant for the period t of the individual's life time. A substitution of this type will make the poverty measure neutral to the income of this person in periods when she did not belong to the population. Now, these extended profiles may be used in place of the original truncated profiles and life time povery level for the individuals may be computed in the usual fashion.[7] We would argue, therefore, that the results in this paper have wider application than to simply poverty and mortality.

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Appendix 1: Definitions

Continuity (C): For all $n \in Z_+$, $S \in R_+^\infty$ and $T \in H$, $P(A,T,S,n)$ is a continuous function of all elements $y_{i,k}$, $k = 1,...,l_i$ and $i = 1,...,n$, in A.

Focus (F): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and $B = (X_1, X_2, ..., X_n)$ with elements $y_{i,k} = x_{i,k}$ whenever $y_{i,k} < z_{t_i+k}$ and $x_{i,k} < z_{t_i+k}$, we have $P(A,T,S,n) = P(B,T,S,n)$.

Monotonicity (M): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and $B = (X_1, X_2, ..., X_n)$ with elements $y_{i,k} = x_{i,k} < z_{t_i+k}$ for all $i \neq j$, $y_{j,k} = x_{j,k} < z_{t_j+k}$ for all $k \neq l$ and $y_{j,l} < x_{j,l} < z_{t_j+l}$, we have $P(A,T,S,n) > P(B,T,S,n)$.

Symmetry (S): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, if $B = (Y_{\pi(1)}, Y_{\pi(2)}, ..., Y_{\pi(n)})$ and $T' = (t_{\pi(1)}, t_{\pi(2)}, ..., t_{\pi(n)})$, where $(\pi(1), \pi(2), ..., \pi(n))$ is any permutation of $(1, 2, ..., n)$, then $P(A,T,S,n) = P(B,T',S,n)$.

Scale Invariance (SI): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profile $A = (Y_1, Y_2, ..., Y_n)$, consider a new income profile $A' = (X_1, ..., X_n)$ and a new subsistence requirement vector $S' = \{..., z'_0, ...\}$ such that $\frac{x_{i,t}}{z'_{t_i+t}} = \frac{y_{i,t}}{z_{t_i+t}}$ for all i and for all t. Then $P(A,T,S,n) = P(A',T,S',n)$.

Translation Invariance (TI): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, consider a new income profile $A' = (X_1, ..., X_n)$ and a new subsistence requirement vector $S' = \{..., z'_0, ...\}$ such that $x_{i,t} - z'_{t_i+t} = y_{i,t} - z_{t_i+t}$ for all i and for all t. Then $P(A,T,S,n) = P(A',T,S',n)$.

Population Principle (P): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, if $B = (Y_1, ..., Y_1, Y_2, ..., Y_2, ..., Y_n, ..., Y_n)$ is a m-fold replication of A and T' is the corresponding m-fold replication of T, then $P(A,T,S,n) = P(B,T',S,mn)$.

Interpersonal Transfers Principle (TR): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$, if another population income profile B is given by $(Y_1, ..., Y_{i-1}, X_i, Y_{i+1}, ..., Y_{j-1}, X_j, Y_{j+1}, ..., Y_n)$ such that $y_{i,k} = x_{i,k}$ for $k \neq l$ and $y_{j,k} = x_{j,k}$ for $k \neq t = l + (t_i - t_j)$, and $0 < y_{i,l} < x_{i,l} = y_{i,l} + \delta < y_{j,t} - \delta = x_{j,t} < y_{j,t} < z_{t_i+l}$ then $P(A,T,S,n) > P(B,T,S,n)$.

Subgroup Decomposability (D): There exists $Q : R_+ \times R_+ \to R_+$ such that, for all $n \in Z_+$, $S \in R_+^\infty$ and $T \in H$, if we partition any population income profile $A = (Y_1, Y_2, ..., Y_n)$ into two such matrices $A^1 = (Y_1, Y_2, ..., Y_{n_1})$ and $A^2 =$

$(Y_{n_1+1}, Y_{n_1+2}, ..., Y_n)$, where $1 \leq n_1 \leq n$ and the birth date vector T in to $T^1 = (t_1, ..., t_{n_1})$ and $T^2 = (t_{n_1+1}, ..., t_n)$ in a similar fashion, then

$$P(A, T, S, n) = Q(P(A^1, T^1, S, n_1), P(A^2, T^2, S, (n - n_1))).$$

The function Q may be regarded as an aggregate deprivation function where deprivation may be measured in terms of relative or absolute shortfall of income in each period from the subsistence requirement. We can also view this in terms of the censored income profiles. (For a detailed discussion on similar properties related to the usual static, or one period, poverty measurement see **Zheng, 1997**).

Intertemporal Symmetry (ITS): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and for any $l, k \in Z$ such that $t_i < l < k < t_i + l_i$ for all $i = 1, ..., n$, if $A' = (Y_1', Y_2', ..., Y_n')$ with $Y_i' = (y_{i,1}, ..., y_{i,l-t_i-1}, y_{i,k-t_i}, y_{i,l-t_i+1}, ..., y_{i,k-t_i-1}, y_{i,l-t_i}, y_{i,k-t_i+1}, ..., y_{i,l_i})$ and $S' = (..., z_{l-1}, z_k, z_{l+1}, ..., z_{k-1}, z_l, z_{k+1}, ...)$, then $P(A, T, S, n) = P(A', T, S', n)$.

Intertemporal Consistency (ITC): For all $n \in Z_+$, $S \in R_+^\infty$, $T \in H$; for any $1 \leq i \leq n$ and for population income profiles $A = (Y_1, Y_2, ..., Y_n)$ and $B = (Y_1, Y_2, ..., Y_{i-1}, X_i, Y_{i+1}, ..., Y_n)$ where $Y_j = (y_{j,1}, ..., y_{j,l_j})$ for j = 1,...,n and $X_i = (x_{i,1}, ..., x_{i,l_i})$, consider the following.

For any $1 \leq l \leq l_i$, define $A_1 = (Y_1, Y_2, ..., Y_{i-1}, Y_i^1, Y_{i+1}, ..., Y_n)$ and $A_2 = (Y_1, Y_2, ..., Y_{i-1}, Y_i^2, Y_{i+1}, ..., Y_n)$ where $Y_i^1 = (y_{i,1}, ..., y_{i,l})$ and $Y_i^2 = (y_{i,l+1}, ..., y_{i,l_i})$. Similarly define $B_1 = (Y_1, Y_2, ..., Y_{i-1}, X_i^1, Y_{i+1}, ..., Y_n)$ and $B_2 = (Y_1, Y_2, ..., Y_{i-1}, X_i^2, Y_{i+1}, ..., Y_n)$ where $X_i^1 = (x_{i,1}, ..., x_{i,l})$ and $X_i^2 = (x_{i,l+1}, ..., x_{i,l_i})$. Also define $T_2 = (t_1, ..., t_{i-1}, l, t_{i+1}, ..., t_n)$.

Then, $P(A, T, S, n) > P(B, T, S, n)$ whenever $P(A_1, T, S, n) > P(B_1, T, S, n)$ and $P(A_2, T_2, S, n) = P(B_2, T_2, S, n)$.

Appendix 2: Proofs

Proof of Theorem 1: The first part of the proof of this result is similar to that of Proposition 1 and 3 in **Tsui (2002)**, hence, we only present an outline of it. First of all, note that the sufficiency part of this result is very easy to verify. For the necessity part, we proceed as follows.

By property (F), we can redefine the poverty measure $P(.)$, in (1), on the censored income profiles Y_i*. Property (D) and (S) together implies that the aggregate measure $P(.)$ can be written as an aggregate of individual poverty levels $\sum_{i=1}^{n} \phi^1(Y_i*, t_i, S, n)$, where the identical functional form $\phi^1(.)$ is due to (S). We now invoke (P) to arrive at the average form given by

$$\frac{1}{n}\sum_{i=1}^{n} \phi^2(Y_i*, t_i, S)$$

where $\phi^2 : \Omega \times Z \times \Omega \to R_+$ is continuous, decreasing and strictly convex in each of the arguments $y_{i,k}*$, $k = 1, ..., l_i$ of Y_i*. The continuity and decreasingness of $\phi^2(.)$ follows from (C) and (M). Finally, convexity of $\phi^2(.)$ in each argument follows from (TR).

Now, let us look at the function ϕ^2 more closely. The arguments t_i are relevant for the poverty calculations only so far as they indicate which element of S to link with any argument of Y_i*. Thus, one can suitably redefine the ϕ^2 function to

$$\phi(Y_i*; z_{t_i+1}, ..., z_{t_i+l_i}) : \Omega \times \Omega \to R_+$$

with similar properties. Note that, the influence of the terms $y_{i,k}$ and z_l on ϕ are independent for $l \neq t_i + k$. So, we may take such terms to be separable in ϕ. For similar reasons, the different z_{t_i+k} terms will also be separable.

Proof of Theorem 2: (a) Define u as a subsistence requirement vector all of whose entries equal one. Let $R = (R_1, ..., R_n)$ where R_i's are as defined in the statement. Now note that, due to (SI), $P(A, T, S, n) = P(R, T, u, n)$

We now invoke the other axioms as in Theorem 1 to arrive at the form

$$\frac{1}{n}\sum_{i=1}^{n} \phi(R_i; 1, ..., 1)$$

with the desired properties. Now, this can be redefined as equation (3). This demonstrates the necessity part. Sufficiency can be easily verified by checking that the class of poverty measures given by (3) satisfies all the assumptions.

(b) Again, if we define the subsistence requirement vector O, all of whose entries equal 0, we can similarly show that $P(A, T, S, n) = P(A', T, O, n)$ where $A' = (A_1, ..., A_n)$.

We now invoke the other axioms as in Theorem 1 to arrive at the form

$$\frac{1}{n}\sum_{i=1}^{n}\phi(A_i; 0, ..., 0)$$

with the desired properties. Now, this can be redefined as equation (4). Hence the necessity. Sufficiency is once again easy to check.

Proof of Theorem 3: The proof proceeds in the same way as that of Theorem 1 and 2. Only now we look at the extended income profiles $\widehat{Y_i}$ for the i^{th} individual. Now individual level normative life time poverty may be redefined as $\phi_R^e(\widehat{R}_i)$ or $\phi_A^e(\widehat{A}_i)$ (depending on whether (SI) or (TI) is invoked), where $\widehat{R}_i = (\widehat{r}_{i,1}, ..., \widehat{r}_{i,L})$ with $\widehat{r}_{i,t} = \widehat{y}_{i,t}*/z_{t_i+t}$ and $\widehat{A}_i = (\widehat{a}_{i,1}, ..., \widehat{a}_{i,L})$ with $\widehat{a}_{i,t} = z_{t_i+t} - \widehat{y}_{i,t}*$ for $t = 1, ..., L$ and $i = 1, ..., n$.

Proof of Theorem 4: (a) In view of theorem 3(a), we start with the form (6) and invoke (ITC) on $\phi_R(R_i)$, this is analogous to imposing (SC) on interpersonal snapshot poverty measures. Proceeding as in **Foster and Shorrocks (1991, p. 693 - 4)** we arrive at the form of $\phi_R^e(\widehat{R_i})$ given by

$$G^e(\frac{1}{L}\sum_{t=1}^{L}\psi_{i,t}^R(\widehat{r_{i,t}})).$$

Now the axiom (ITS) imposes symmetry between the functions $\psi_{i,t}^R$ for each t. So the functions $\psi_{i,t}^R$ becomes independent of t and, due to (S), may only depend on L, which is a population specific parameter. Thus we finally arrive at the form $\psi_R^e(\widehat{r_{i,t}})$ for each $\psi_{i,t}^R(\widehat{r_{i,t}})$. Continuity of the functions G^e and ψ_R^e follow from (C). Increasingness of G^e and decreasingness of ψ_R^e follow due to (M). Finally, (TR) implies convexity of the functions under consideration. Hence, the aggregate poverty measure reduces to the form (8). Sufficiency can be easily verified by checking that the class of poverty measure given by (8) satisfies all the assumptions.

(b) The proof of this part is similar to that of part (a), given theorem 3(b), and we omit the details.