A NOTE ON ESTIMATING TAX ELASTICITIES

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<u>ABSTRACT</u>

The most popular technique for estimating tax elasticities is the "Proportional Adjustment" method. This paper shows that the standard methodology used will almost invariably lead to biased elasticity estimates, and proposes an alternative methodology which avoids this problem.

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INTRODUCTION

Possibly one of the commonest and most important empirical issues in applied Public Finance is to estimate the likely behaviour of tax receipts in relation to changes in the tax base. Such estimation is essential not only for purposes of formulating government budgets and monitoring the progress of tax collections, but also for a variety of other research applications. In particular, almost any macro-economic modeling exercise requires the specification of tax functions. Thus, in the Indian context, the national and state-level Five Year Plans and the awards of successive Finance Commissions have been based on such estimates.

Conceptually, the most appropriate measure of the responsiveness of tax revenues to changes in the base for most analytical applications is the 'elasticity' or, in the words of A.R. Prest, the "built-in flexibility",[2] which seeks to relate the percentage change in tax revenue to a percentage change in the tax base with a **given** tax structure. However, since legislative changes in the tax structure alter this relationship from time to time, direct measurement of the tax elasticity from a historical revenue series often becomes problematic. The problem becomes even more complex if the tax base itself is not precisely measurable or if such data are not available and recourse has to be taken to using proxy measures. This is in fact a very common problem since most analytical studies on tax responsiveness tend to deal with broad categories of taxes, which are aggregates of a wide variety of tax rates applied to different tax bases.

In estimating the built-in elasticity of a tax, therefore, either the time series data on tax revenues need to be adjusted to eliminate the effects of discretionary tax measures, or a suitable estimation methodology has to be adopted, or a combination of the two. The most appropriate method would clearly depend upon the availability, nature and reliability of information on tax revenues, discretionary changes in the tax structure and tax bases. Over the years, at least four approaches have been used :
- (1) proportional adjustment;
- (2) constant rate structure;
- (3) Divisia index; and
- (4) econometric methods.

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[2] Prest (1962).

Of these, the constant rate structure method, which involves the generation of a simulated tax revenue series on the basis of the effective tax rate for a given reference year and estimates of the tax base for subsequent years, is clearly the most accurate provided that both the tax and its base are defined narrowly enough to permit application of the reference year rates to later year tax bases with a certain degree of confidence.[3] It is evident, however, that such a procedure will usually be extremely cumbersome if it is applied to the full range of tax instruments that exists in any country, and that its data requirements are necessarily very heavy indeed. As a consequence, the constant rate structure method is rarely used for analytical purposes, and is normally relevant only when substantial changes are being considered in the tax structure.[4]

For most analytical work, therefore, recourse is taken to one of the other three methods. Of these, the Divisia index and the econometric methods are least demanding in terms of data requirements, since they rely mainly on actual tax collections and tax base measures at fairly aggregative levels. Nevertheless, they are both subject to certain weaknesses which need to be noted. As far as the Divisia index is concerned, its computation is predicated on the conditions that the underlying tax function is continuously differentiable and homogeneous, preferably linear homogeneous.[5] Although these may not seem to be particularly demanding conditions, there are serious doubts about their validity when the aggregate tax to which it is being applied comprises of a non-constant set of items on which taxes are being levied. If the estimation is being done over a sufficiently long period of time, experience shows that for most countries, especially developing countries, the composition of the tax base will exhibit significant change.

The econometric models, which rely mainly on using dummy variables to capture discretionary changes in tax rates and tax structures, cannot be used if discretionary tax changes have been made frequently in the past, since it leads to an excessive reduction in the degrees of freedom and thereby to the efficiency of the estimators. Even if the number of such discretionary changes is relatively small, serious problems can arise in the specification of the estimation equations unless there is information on the nature of the tax changes and the extent to which their effects are independent of one another.

The proportional adjustment method falls somewhere in between in terms of its data requirements. While, on the one hand, it does not require

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[3] For instance, this method cannot be applied to broad tax categories such as excise or customs, but to individual products within these categories.

[4] This method is useful for instances in cases where revenue-neutral tax simplifications are being worked out.

[5] See Hulten (1973)

disaggregated data on tax rates and tax bases, which are necessary for the constant rate structure method; it cannot, on the other hand, make do only with actual tax collection data as is possible with the Divisia index method. It requires the use of budget estimates of tax yield arising out of discretionary changes. Such data are often not available in many countries, which restricts the applicability of this method. Nevertheless, if such data are available, this method yields better estimates of tax elasticity than either the Divisia index or the econometric methods.[6]

In the Indian case, estimates of tax yields arising out of discretionary changes in tax rates and coverages are routinely available in the budget documents. Therefore, the application of the proportional adjustment method is perfectly feasible for estimating tax elasticities in India. There have been several such attempts,[7] but the weight of general opinion is that these estimates are not particularly accurate, primarily because of the questionable reliability of the budget estimates of the effects of the discretionary changes. This judgment is based primarily on comparisons between the predicted and the actual tax collections for in-sample forecasts.

The net result of this dissatisfaction with the methodology has been that, in recent years, the use of elasticity estimates in forecasting tax collections has all but ceased in India, and recourse is increasingly being taken to the use of buoyancy estimates for most analytical purposes.[8] This is unfortunate, since the use of buoyancies in making forecasts or projections implicitly assumes that there is a well-defined trend in the discretionary changes that have been made in the past, and that this trend will continue in the future as well. In other words, it completely ignores the policy dimension of any change in the tax structure, and imbues it with an almost behavioural attribute. As a result, large potential errors are introduced in the projections, which completely negate their use not only in analytical work but also for monitoring tax compliance and administration.

The purpose of this paper is to suggest that the observed errors in projection of tax revenues by use of the proportional adjustment method may arise from the methodology itself, especially its data cleaning procedure, and not necessarily as a consequence of unreliable budget estimates. An alternative data cleaning procedure is also developed, which addresses the inherent weakness of the existing methodology through more complete utilisation of the available data.

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[6] See Choudhry (1979) and Gillani (1986)

[7] The first such attempt is Sahota (1961), although his methodology is not commonly referred to in this context.

[8] Buoyancy is defined as the percentage change in tax revenue to a percentage change in the tax base without any correction for changes in the tax structure. It therefore measures the combined effect of the change in the tax base and the discretionary changes in tax rates.

PROPORTIONAL ADJUSTMENT : A CRITIQUE

The proportional adjustment method for computing tax elasticities involves a three-step process.[9] In the first stage, a preliminary series of adjusted tax yields is obtained by subtracting from the actual yield the budgetary estimates of the effects of discretionary tax changes.[10] In the second step, this preliminary series is further adjusted to exclude the continuing impact of each discretionary change on all future years' tax yields by multiplying by the ratio of the previous year's adjusted figure to the actual tax receipt. It can be shown that this procedure involves a factor sequence, each element of which represents the effect of the automatic component of tax changes in earlier years. These two steps constitute the 'data cleaning' process. In the third step, the resulting series of 'cleaned' tax yields is then regressed on some measure of the tax base to obtain the necessary elasticity values.

The essential weakness of the proportional adjustment method lies in the data cleaning procedure. It is asserted that this procedure yields a series which is systematically biased, and will therefore lead to biased elasticity estimates. Before entering into a demonstration of the nature and cause of this bias, it may be desirable to first specify the proportional adjustment data cleaning procedure more precisely. Notationally, the data cleaning process may be described in the following manner:

Let :
AT_i = the adjusted or cleaned tax yield in year i
T_i = the actual tax yield in year i
D_i = budget estimate of the yield arising out of discretionary tax changes in year i

In the reference year '0', i.e. the year whose tax structure is to be used as the basis for building up the adjusted series, the adjusted tax yield is set at the actual:

$$AT_0 = T_0 \qquad\qquad\qquad (1)$$

For the following year :

$$AT_1 = T_1 - D_1 \qquad\qquad\qquad (2)$$

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[9] There are alternative methods for doing proportional adjustment. Although its genesis is in a seminal paper by Prest (1962), the method described here is the most popular one, which is based on the procedure proposed by Mansfield (1972).

[10] Discretionary changes are defined as legal changes in the tax rates or in the tax base, introduction of new taxes, and certain specific changes in tax effort. Thus the effects of normal changes in tax effort or in tax administration are treated as a part of the normal tax yield.

Since AT_0 is equal to T_0 by equation (1), no further adjustment is needed. In every subsequent year, however, the non-discretionary component of tax receipts have to be adjusted in the following manner:

$$AT_j = (T_j - D_j)\frac{AT_{j-1}}{T_{j-1}} \quad \forall \; j = 2, \ldots, n \tag{3}$$

Through sequential substitution it can be shown that equation (3) can be rewritten as :

$$AT_j = AT_1 \cdot \prod_{i=2}^{j} \frac{(T_i - D_i)}{T_{i-1}} \quad \forall \; j = 2, \ldots, n \tag{4}$$

which is in essence the Mansfield equation for proportional adjustment data cleaning.

In order to appreciate the bias that is introduced in the adjusted series by this data cleaning methodology, it is useful to benchmark it against an assumed tax function. For this purpose, consider the simplest of tax functions:

$$T_t = t_t \cdot B_t \tag{5}$$
where: t_t = tax rate at time t
B_t = tax base at time t

Clearly, the ideal adjusted series for estimating the elasticity of the tax function (5) should be as follows:[11]

$$AT_t = t_0 \cdot B_t \quad \forall \; t = 0, \ldots, n \tag{6}$$

In order to derive the equivalent proportionately adjusted tax series by using the Mansfield method as given by equations (1) and (4), it is assumed that the discretionary changes in the tax rate (Δt_t) are known with certainty and the only uncertainty is associated with the base (B_t).[12] Thus, the tax authorities provide the budgetary estimate of the discretionary changes in the tax rate by multiplying the change in the tax rate with an estimate of the base (B_t^e) for the coming fiscal year. Thus:

$$D_t = \Delta t_t \cdot B_t^e \tag{7}$$

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[11] The tax function (5) yields a unit tax elasticity, which will be empirically obtained only when the adjusted series takes the given structure.

[12] With more complex tax structures, there could be uncertainty associated with changes in the effective tax rates as well. This would complicate the presentation further, and is therefore being ignored for expository purposes. It should be noted, however, that in such cases, the argument being made here gets further strengthened.

Using equation (7), the second term of the proportionally adjusted series is:[13]

$$AT_1 = T_1 - D_1 = t_1.B_1 - \Delta t_1.B_1^e \qquad (8)$$

Since there is no uncertainty regarding the tax rate:[14]

$$t_1 = t_0 + \Delta t_1 \qquad (9)$$

Substituting equation (9) into equation (8) yields:

$$AT_1 = t_0.B_1 + \Delta t_1.(B_1 - B_1^e) \qquad (10)$$

As can be seen, a discrepancy between the ideal series given by equation (6) and the proportionally adjusted series appears from the second term itself, which arises out of any difference between the tax base estimated at the beginning of the year and the actual. The problem gets further compounded in every subsequent year as the proportional adjustments are made. In the second year, for instance:

$$AT_2 = (T_2 - D_2)\frac{AT_1}{T_1} = \frac{(t_2 - \Delta t_2.B_2^e)\{t_0 B_1 + \Delta t_1.(B_1 - B_1^e)\}}{t_1 B_1} \qquad (11)$$

$$= t_0 B_2 + \frac{t_0}{t_1}.\Delta t_2.(B_2 - B_2^e) + \frac{B_2}{B_1}\Delta t_1(B_1 - B_1^e) + \frac{\Delta t_1.\Delta t_2.(B_1 - B_1^e)(B_2 - B_2^e)}{t_1 B_1}$$

As should be obvious, the sequential adjustments that are made in the cleaning process leads to a situation in which the right hand side of the proportionally adjusted series in any given year 'n' will contain 2^n terms of progressively higher order, the first of which will be $t_0.B_n$ – the corresponding term of the ideal adjusted series – and the rest representing deviations from the ideal series. In general, the proportionally adjusted series will take the form:

$$AT_n = t_0.B_n + \sum_{i=1}^{n}\Delta t_i.\frac{t_0}{t_{i-1}}.(B_i - B_i^e)\frac{B_n}{B_i} + \;\;...... \text{ higher order terms} \qquad (12)$$

Even if it is assumed that the higher order terms are relatively small in magnitude, and therefore insignificant in terms of their impact on the extent of

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[13] The first term, by definition, is the same as in the ideal adjusted series, i.e. $AT_0 = t_0.B_0$.

[14] In general, it is assumed that : $t_t = t_{t-1} + \Delta t_t$

discrepancy from the ideal,[15] it should be evident from equation (12) that the second order terms alone can introduce sizable bias in the proportionally adjusted series. More importantly, these biases are by no means random since they are directly related to B_n. Thus, if AT_t is regressed on B_t, as is necessary for deriving the tax elasticity, these terms will introduce a systematic bias in the parameter estimate.

In short, therefore, the proportional adjustment method, as commonly used, will almost always yield biased estimates of the tax elasticity. The source of this bias no doubt lies in faulty budget estimates of the discretionary tax changes,[16] but is due in at least equal measure to the inability of the methodology to make corrections for these errors. Since budget estimates are, by their very nature, based on projections for the coming financial year, it would be too much to expect the tax authorities to be consistently accurate in their forecasts of variables which are not in their control. Of course, it would be sufficient if the forecast errors were randomly distributed, but even this is too much to hope for. It would be preferable to develop a methodology which explicitly takes into account the possibility that projection errors will be made and attempts to correct for them by using additional information.

AN ALTERNATIVE METHODOLOGY FOR DATA CLEANING

Once it is recognised that inaccurate and biased estimates of tax elasticities arise primarily out of projection errors made by the tax authorities while computing the effect of discretionary tax changes, and that such errors are inevitable in any projection exercise, it is not difficult to identify a fairly obvious and intuitively attractive method of correcting the estimates. Budget documents invariably provide estimates of expected revenues from each tax, inclusive of the discretionary component, and not just of the effect of the discretionary tax changes.[17] Therefore, if it can be assumed that the two estimates are made on the basis of the same projections, then it should be possible to calibrate the estimate of the non-discretionary component of tax receipts in each year by using the ratio of the actual to the estimated total tax receipts. Having done so, there is of course the need to exclude the continuing impact of every discretionary change in the future years, for which the second step of the proportional adjustment method can continue to be used. Thus, the proposed method is a

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[15] This is actually a very strong assumption since it can be shown that the higher order terms can be of substantial magnitude, particularly if discretionary changes to the tax structure have been carried out fairly often. Readers are invited to verify this assertion for themselves.

[16] It may be noted that if $B_t^e = B_t$ always, the proportional adjustment method will yield the ideal series.

[17] In fact, the Indian budget documents provide estimates of the total tax receipts and of the receipts that would have accrued without the discretionary change. Thus, the effect of the discretionary change is calculated as the difference between the two.

variant of the proportional adjustment method, which makes more complete use of the available data in order to address the inherent problem of the standard proportional adjustment data cleaning methodology.[18]

Notationally, the proposed alternative data cleaning process is as follows:

Let T_i^e = budget estimate of the tax receipt inclusive of any discretionary change in year i

In the reference year, as earlier :

$$AT_0 = T_0 \tag{13}$$

In the following year, however, the formulation is different:

$$AT_1 = \frac{(T_1^e - D_1).T_1}{T_1^e} \tag{14}$$

In every subsequent year:

$$AT_i = (T_i^e - D_i).\frac{T_i}{T_i^e}.\frac{AT_{i-1}}{T_{i-1}} \quad \forall \ i = 2, ... , n \tag{15}$$

Through sequential substitution it can be shown that equations (14) and (15) can be rewritten as:

$$AT_j = T_j.\prod_{i=1}^{j}\frac{(T_i^e - D_i)}{T_i^e} \quad \forall \ j = 1, ... , n \tag{16}$$

which is the analogue of the Mansfield equation (4) for the modified proportional adjustment method.

The first point that needs to be noted is that if there are no projection errors in the budget estimates of total tax receipts; i.e. if $T_i^e = T_i \ \forall \ i$, then both equations (4) and (16) reduce to an identical expression.[19] In other words, the modified proportional adjustment method becomes relevant only when it is expected that there are significant estimation errors – which is probably most of

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[18] In popular terminology, the standard proportionally adjusted series is referred to as "Prest cleaned", after A.R. Prest who originally developed the methodology. For the lack of a better term, it is suggested that the revised methodology being proposed in this paper could be called : "Washed, Prest cleaned".

[19] It can be shown that in such a situation, both equations (4) and (16) become:

$$AT_n = (T_n - D_n).\frac{(T_{n-1} - D_{n-1})}{T_{n-1}}......\frac{(T_1 - D_1)}{T_1}$$

the time. It is, however, necessary to demonstrate that the modified method will yield better results than the standard in the presence of estimation errors. In order to do so, it would be useful to compare the adjusted series arising out the modified data cleaning process applied to the benchmark tax function (5) with the ideal adjusted series given by equation (6).

As earlier, the first term in the adjusted series is definitionally the same as that of the ideal. The second term, however, is as follows:

$$AT_1 = (T_1^e - D_1).\frac{T_1}{T_1^e} \tag{17}$$

Since:

$$T_1^e = t_1.B_1^e \; ; \tag{18}$$

$$D_1 = \Delta t_1.B_1^e \; ; \text{ and} \tag{19}$$

$$t_1 = t_0 + \Delta t_1 \tag{20}$$

substituting equations (18), (19) and (20) into (17) yields:

$$AT_1 = \left[(t_0 + \Delta t_1).B_1^e - \Delta t_1.B_1^e\right]\frac{t_1.B_1}{t_1.B_1^e} \tag{21}$$

collecting terms and canceling leaves:

$$AT_1 = t_0.B_1 \tag{22}$$

A comparison of equation (22) with the corresponding term of the standard proportional adjustment series given by equation (10) shows that the modified method does not introduce an error term right from the outset, and is equivalent to the corresponding term in the ideal series. The question is whether the same characteristic obtains for the later terms as well. Consider then the third term in the modified cleaned series:

$$AT_2 = (T_2^e - D_2).\frac{T_2}{T_2^e}.\frac{AT_1}{T_1} \tag{23}$$

substituting terms yields:

$$AT_2 = [(t_1 + \Delta t_2)B_2^e - \Delta t_2.B_2^e].\frac{t_2 B_2}{t_2 B_2^e}.\frac{t_0 B_1}{t_1 B_1} \tag{24}$$

Canceling terms in equation (24) gives the final expression of the third term of the modified cleaned series as:

$$AT_2 = t_0.B_2 \qquad\qquad (25)$$

which is again the same as that of the ideal cleaned series. Following the same procedure as above, it can be shown that the modified cleaning methodology described by equation (16) yields the ideal cleaned series for all terms, at least for this extremely simple tax function.[20] Thus the use of this series in estimating the tax elasticity will not give rise to biased estimates, unlike in the case of he standard proportional adjustment method. The reason for this is that the calibration procedure used in the proposed cleaning methodology corrects for the systematic errors in forecasting tax yields.

Therefore, on the basis of the benchmarking that has been carried out, it can unequivocally be asserted that the modified proportional adjustment cleaning process proposed in this paper avoids the inherent bias that exists in the standard cleaning procedure that has been commonly used heretofore; and, therefore, should allay at least some of the apprehensions that exist in using the proportional adjustment method for analytical purposes.

CONCLUDING REMARKS

The principal purpose of this paper was to argue that the dissatisfaction that has often been expressed regarding the use of the proportional adjustment method for estimating tax elasticities arises out of the methodology itself and not from unreliable estimates of discretionary tax changes made by the tax authorities. Errors in estimation are inherent in any projection exercise, and the tax authorities cannot be faulted for such errors. It has also been shown that it is possible to devise an alternate methodology which corrects for such errors by using information which is readily available from budget documents.

There is, moreover, a further point which needs to be made with some force. The proportional adjustment method, especially with the refinements indicated in this paper, combines the best features of the constant rate structure and the econometric methods, and should therefore be ideally suited to estimate the elasticities of broad tax categories, indeed more so than any other method. To elaborate this point, it should be borne in mind that budget estimates of tax yields are usually made by an army of tax officials, each of whom calculates the likely tax receipts from a fairly narrowly-defined tax source, which are then aggregated to derive the total estimated tax yield. The responsibility for making tax assessments is usually broken up not only by the type of tax and by product/income source, but also by geographic region. Thus, every tax official is in effect carrying out an extremely detailed constant rate structure exercise within

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[20] Readers are invited to satisfy themselves of the truth of this assertion.

his own limited domain, except that the rate is being applied to an estimate of the tax base rather than an actual.

Thus, there is every reason to place considerable confidence on the accuracy of these estimates, except to the extent that the projections of the narrowly-defined tax bases diverge from the actuals.[21] In the process of aggregation, however, the estimated micro-level tax yields are added up to derive the total, but usually no effort is made to assess whether the micro-tax bases add up to a reasonable approximation of the aggregate base. This introduces yet another source of divergence between the estimate made of the base and the actual, which is compounded by the fact that the former is usually unobservable. The proposed data cleaning methodology is designed to address precisely this problem.

Finally, a few words need to be said about the actual estimation of tax elasticities from the cleaned data set. Normally, a simple log-linear regression function is used for this purpose:

$$\ln AT_t = \delta_0 + \delta_1 . \ln X_t \tag{26}$$

where: X_t = some aggregative measure of the tax base at time t
δ_1 = estimate of the tax elasticity with respect to the base X

There are two problems with this formulation. First, quite often the variable X may not be even a relatively exact analogue of the true base B, and the relationship between the two may vary systematically over time.[22] As a result, the parameter estimate of δ_1 would be influenced by the 'average' relationship between X and B, and not by the trend relationship, which is what would be desired. Second, changes in tax structures can, directly or indirectly, affect the base X itself, which will not be captured by equation (26). Therefore, for these two reasons, it is in general preferable to use "error correction" models to econometrically estimate the tax elasticities. A common formulation is:

$$\ln AT_t = \delta_0 + \delta_1 . \ln X_t + \delta_2 . \ln AT_{t-1} \tag{27}$$

where δ_1 is the estimator of the short-run elasticity, and $\dfrac{\delta_1}{1-\delta_2}$ is the long-run value of the elasticity. Such an estimator should hopefully take care of whatever residual problems exist after the data cleaning has been done.

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[21] It would be too much to assert that tax officials are by and large either incompetent or indulge in deliberate miscalculations.

[22] For instance, the GDP is the most often used base for a variety of tax elasticity calculations where the appropriate bases could be value of industrial output (excise), non-agricultural income (income tax) or corporate profits (corporate tax).

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